ROYAL CANADIAN SERIES.

ARITHMETIC

FOR

PUBLIC SCHOOLS.

TORONTO:
CANADA PUBLISHING COMPANY
(LIMITED)
1882.

PREFACE

THE distinctive purpose of this work has been to furnish a simple, lucid, and systematic exposition of the theory and practice of Arithmetic, covering the ground which is usually traversed by the pupil before entering the High School.

Arithmetic, it may be said, is above everything else a practical study. The art of computation is undoubtedly of much value in the practical concerns of every day life; but the habit of investigating the principles on which this art is based is not of inferior importance. The former gives to the student a mastery of figures, which will be serviceable in commercial and scientific pursuits; the latter tends to concentrate his attention; to induce habits of patient abstraction and accurate thought; to familiarize him with the laws of reasoning, and to lead him to examine carefully the grounds of every inference he draws.

The difficulty hitherto met with in text-books on Arithmetic is that the theoretical has been made subordinate to the practical. But if the theory is imperfectly understood, and the principles are not comprehended, then questions can only be solved mechanically. Hence the necessity of first making the rules intelligible, and then impressing them on the mind by copious and practical illustration. With this object in view, two important principles have been kept in mind: (1) That the exercises shall be so constructed as to require the pupil to think; and (2) That they shall consist largely of examples selected with especial reference to the pursuits of an agricultural and commercial people.

A comparison of the present work with those which are specially valued on account of the character and number of the problems they contain will, it is believed, show that it comprehends everything that is usually regarded as of practical importance in Arithmetic. The prominent features of the work may be enumerated as follows:

(1) The investigation of the principle on which a rule in Arithmetic depends always precedes the statement of the rule itself.

(2) Every process employed in the solution of a question is referred to some general law or axiom in the theory of numbers.

(3) These general truths, as they may be called, are distinctly enunciated and are printed in italics. If self-evident, they are illustrated by simple numerical examples; if otherwise, more extended demonstrations are given: in every case, the truth itself is stated in a clear, concise form.

(4) The solution of money problems, and the application of reduction to concrete quantities in their simplest form, leading the pupil gradually up from the abstract to the concrete, are placed earlier in the course than is usual, and are thus made available in subsequent exercises.

(5) The logical relations of the several parts of Arithmetic are lucidly marked by their arrangement. For example: Reduction is not treated as a separate rule, but so much of it as belongs to Multiplication falls under that head, while the rest takes its proper place as one of the practical applications of Division.

As will be apparent, considerable space has been taken up with exercises for rapid mental work, the importance of which, if the principles which underlie them are fully brought out by the teacher and grasped by the pupil, can hardly be over-estimated. It is confidently believed that the exercises will be found sufficiently numerous and varied, and that the examples solved in Ex. 26, 42, 43, 51, 57, 58 and 59, and in the Examination Papers 1 to 6, will aid in the illustration of the general principles which form the key to all problems in Arithmetic. The work, as a whole, it is hoped, will prove of the highest service to both teacher and student, and merit a permanent place among our Canadian educational text-books.

TORONTO, June, 1882.

CONTENTS.

ARITHMETIC FOR BEGINNERS.

CHAPTER I.

DEFINITIONS AND PRINCIPLES.

NOTATION AND NUMERATION.

1. A Unit is one, or a single thing.

   Ex.—**One**: one boy; one dollar.

2. A Number is a unit, or a collection of units, and answers the question How many?

   Ex.—**Three**, Five, Twelve, Sixty.

3. The unit of any number is one of the collection of units which form the number.

   Ex.—The unit of eight is one; the unit of six horses is one horse; the unit of twenty dollars is one dollar.

4. A Unit may be either **abstract** or **concrete.**

5. An Abstract Unit is one that does not refer to any particular object or thing.

   Ex.—**One.**

6. A Concrete Unit is one that is applied to some particular object or thing.

   Ex.—**One** cent, one quart.

7. A number is Abstract or Concrete, according as its unit is abstract or concrete.

   Ex.—Three, Seven, Twelve, Forty, One Hundred, are Abstract Numbers.

   Three Dollars, **Seven** Gallons, **Twelve** Men, **Forty** Books, **One Hundred** Cents, are Concrete Numbers.

8. All numbers in ordinary use are formed from the characters 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Ex.—384, 9072, 165.

9. The Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are read one, two, three, four, five, six, seven, eight, nine, and are called Digits, or Significant figures.

10. The Figure 0, called nought, zero, or cipher, has no value, and is termed the Insignificant figure.

11. The Art of Expressing Numbers by means of these or other characters is called Notation.

12. Notation is of two kinds, Arabic Notation and Roman Notation.

ARABIC NOTATION.

13. Arabic Notation is the art of expressing numbers by means of the characters 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
(It is so called because it was introduced by the Arabs.)

14. The next number above nine is **ten**, which is expressed by the figure **one** placed to the left of the figure **nought**, meaning **one ten** and **no** units; in the same way **eleven**, or **ten** units and **one** unit, may be expressed by the figure **one** (meaning **one** ten) placed to the left of the figure **one** (meaning **one** unit), and so on for the rest of this order.

15. The complete order will then be :

Ten, represented by	10	Fifteen, represented by	15
Eleven, " "	11	Sixteen, " "	16
Twelve, " "	12	Seventeen " "	17
Thirteen, " "	13	Eighteen, " "	18
Fourteen, " "	14	Nineteen, " "	19

The pupil will notice that the first part of each word agrees with the right-hand figure in the number expressing its value.

16. In like manner, all numbers in the next group are expressed by using the figure **two**, followed by the

original digits in order, the number **twenty** (written 20) meaning **two tens** and **no** units, the rest being formed by proceeding as before, thus:

Twenty-one, represented by 21
Twenty-two, " " 22
Twenty-three, " " 23, etc., etc.

17. The next group will express **three tens**, called thirty, and a certain number of units, the successive numbers being formed in the same manner as before.

18. The remaining groups will commence with the following **key** words:—

Forty,	represented by	40	Seventy,	represented by	70
Fifty,	" "	50	Eighty,	" "	80
Sixty,	" "	60	Ninety,	" "	90

The pupil will again notice the likeness the first part of each word bears to the digit in the number opposite to it.

19. We now come to the greatest number that can be expressed by two figures, viz., 99, meaning, of course, **nine tens** and **nine units.**

EXERCISE 1.

1. Write down the following numbers in figures:—Four, Eleven, Twenty-six, Thirty-seven, Forty-five, Sixty-eight, Seventy-seven, Fifty-nine, Eighty, Ninety-two.

2. Write down the smallest and also the greatest number expressed

by one figure.
by two figures.
by the figures **two** and **three.**
by the figures **three** and **nine.**
by the figures **nine** and **one.**
by the figures **one** and **seven.**
by the figures **seven** and **eight.**
by the figures **eight** and **two.**

3. Write down in order all the numbers

From twenty-five to forty-seven.
" sixty-three to seventy-eight.
" seventeen to eleven.
" forty-seven to thirty-three.

4. Write down the number of words, and also the number of letters, in this sentence.

5. Count the numbers between fifty and seventy-five.

6. Write down any numbers you can make
by using both the figures 6, 8.
by using both the figures 7, 6.
by using both the figures 8, 7.

---

20. We left off at the number 99, and must now show how to represent a number having one more unit than 99 The required number is called one hundred, and is written 100,—the figure 1 meaning one hundred units, the next figure 0 meaning no ten units, and the last 0 meaning no units.

21. We thus arrive at one hundred, two hundred, etc., and by combining the hundreds group, and the preceding group of tens, and the simple group of units, we complete all numbers that can be formed with three figures.

Ex.—One hundred and sixty-four is written 164, meaning one hundred units, six tens of units (or sixty units), and four units.

Four hundred and four would be written 404, the zero meaning that there are no units in the tens group.

Eight hundred and sixty is written 860, the zero implying that there are no units in the units group.

22. We now see the importance of the figure 0 as a means of keeping other figures in their places, for without it 404 would be 44, four tens and four units; 860 would be 86, or eight tens and six units.

23. The hundreds group or order brings us up to the number 999, the largest that can be written with three figures.

Ex.—Write down the number Three Hundred and Seventy-nine.

The three, standing for hundreds, must be placed on the left of the required number, in the hundreds'

place; the **seventy** or, **seven** tens, we must put in the next place, to the right of the **three**—that is, in the tens' place; and the **nine**, which means just nine units, must be in the next or units' place. The number then stands thus:—

Hundreds.	Tens.	Units.
3	7	9

or, 379.

If the number had been three hundred and nine, then, there being **no tens**, the tens' place would be filled by **zero**, as 309. If the number had been three hundred and seventy, then, there being no units, the units' place would be filled by the **zero**, as 370.

The number 083, meaning **no** hundreds and **eight** tens and **three** units, should be written **83**, for it can be of no use to express hundreds when there are none.

EXERCISE 2.

1. Write down the following numbers:—One hundred and seventeen; three hundred and eleven; five hundred and eleven; five hundred and seventy-five; eight hundred and ninety-nine; four hundred; sixty-nine; five hundred and seven; eight hundred and sixty; four hundred and ten; nine hundred and nine; seven hundred and eighty-seven.
2. Write down, in order, the numbers between **one hundred and five** and **one hundred and twenty.**
3. Write down all the numbers of three figures in which the two left hand figures are **seven** and **nine**; in which the two right hand figures are **eight** and **nought**; in which the outside figures are **six** and **seven**; in which the outside figures are **seven** and **nought.**
4. Write down the greatest and least number composed of three figures.
5. Write down all the numbers formed from the figures **six, eight** and **nine.**

6. Write down, in order, all the numbers made up of the figures **three, nought, nine,** commencing with the smallest number.

7. How many numbers are there between **one hundred** and **one hundred and ninety-nine**; between **ninety-seven** and **one hundred and nine.**

8. Write down, in order, all the different numbers formed by using

(1)	the	three	figures	7, 0, 0
(2)	"	"	"	8, 0, 6
(3)	"	"	"	9, 9, 8
(4)	"	"	"	7, 7, 7

---

24. We next proceed to numbers of four figures, the first one after 999 being written 1000, and called **one thousand.** The remaining numbers of four figures are formed by writing **thousands** in the **fourth** place, **hundreds** in the **third** place, **tens** in the **second** place, and **units** in the **first** place from the right. Thus 7809 represents **nine** units of the **first** order, **no** units of the **second** order or **tens**, eight units of the **third** order or **hundreds**, and seven units of the **fourth** order or **thousands.** The whole number is read seven thousand eight hundred and nine.

25. The pupil will by this time see that **one** unit of any of these orders has the same value as **ten** units of the next lower order, that is to say:—**One** thousand is the same in value as **ten** hundreds; **one** hundred is the same as **ten** tens; and **one** ten the same as **ten** units.

26. We should therefore naturally expect to meet after the order of thousands a fifth order representing **ten** thousands, and this is the case; but instead of giving it a new name it is known as the order of **tens** of **thousands.** Thus 86079 represents **eight** tens of thousands, **six** thousands, **no** hundreds, **seven** tens, and **nine** units: or eighty-six thousand and **seventy-nine.**

27. As the order **hundreds** followed in value the order **tens** (Art. 20), so now we have **hundreds of thousands** following in value **tens of thousands**, and forming the sixth order from the right.

28. Proceeding as before, we obtain (after one hundred thousand), two hundred thousand, three hundred thousand, and so on until we reach ten hundred thousand. This is expressed by a new name, and called a **million**.

29. After counting to a **million**, we proceed to count **one** million, **two** millions, **three** millions, and so on as far as **ten** millions, which forms the seventh place from the right, or the seventh order.

30. After this **hundreds of millions** will follow **tens** of **millions**, just as **hundreds of thousands** followed **tens of thousands** (Art. 27); and **hundreds** followed **tens** (Art. 20). We then come to **ten hundreds of millions**, which is called a **billion**.

31. The figures 1, 2, 3, 4, etc., when they stand alone, or when they occupy the first place, denote simply so many **units** or **ones**, and are called units of the **first order**. When they occupy the **second place**, they represent **tens**, and are called units of the **second order**. When found in the **third** place, they stand for **hundreds**, and are called units of the **third order**, and so on. This may be illustrated by the following table:—

The	1st	order	of units	is called **units**.
"	2nd	"	"	" **tens.**
"	3rd	"	"	" **hundreds.**
"	4th	"	"	" **thousands.**
"	5th	"	"	" **ten-thousands.**
"	6th	"	"	" **hundred-thousands.**
"	7th	"	"	" **millions.**
"	8th	"	"	" **ten-millions.**
"	9th	"	"	" **hundred-millions.**
"	10th	"	"	" **billions.**
"	11th	"	"	" **ten-billions.**
"	12th	"	"	" **hundred-billions.**

And we may extend this table to **trillions, quadrillions,** etc.

32. The pupil must now practice writing down on the slate any number that may be read.

Ex.—Express in figures the number nine hundred and seventy thousand six hundred and eight.

Here the highest order of units is **hundreds of thousands**, and there are **nine** of them; the next order is **tens of thousands**, of which there are **seven**; the next order is **thousands**, of which there are **none**; the next **hundreds**, of which we have **six**; the next **tens**, of which there are none; the next **units**, of which we have **eight**. To write the number we first set down the units of the highest order, and this is **hundreds of thousands**; then the **tens of thousands, thousands, hundreds, tens, units**, one after the other, always putting in a **cipher** or **zero** for the order of which there are **no** units. The number must then be written

970608.

☞ Too much attention cannot be paid to the placing of the zero.

EXERCISE 8.

1. Write the following numbers in figures:

Nine thousand and forty-eight.
Five thousand and seven.
Forty-three thousand six hundred and fifty-nine.
Five hundred and thirty-six thousand three hundred and two.
Four hundred and five thousand three hundred and thirteen.
Five million forty-three thousand and thirty-seven.
Sixty-four thousand seven hundred and ninety-two.
Three hundred and fifty-six thousand and ninety-seven.
Nine million three hundred and forty-five thousand and twenty-seven.
Eighty thousand and fifty-six.
Nine million ninety thousand nine hundred.
Eighty-three thousand and seven.
Nine thousand and ninety.

Write he following numbers in figures:

Three hundred and seventy-five million eight hundred and sixty-seven thousand seven hundred and ninety-nine.

Eleven thousand and seventy-one.

Six million eight thousand seven hundred and four.

2. Write down the greatest and least numbers that can be formed

by using 4 figures.
" " 5 "
" " 6 "
" " 7 "

3. Write down, in order, all the numbers of four figures having 3 for the left hand figure and 72 for the two right hand figures.

4. Write down the greatest and least numbers that can be formed by using all the figures 7, 3, 0, 5.

5. Write down all the different numbers that can be formed by using all the figures 7, 0, 0, 8, and name the greatest and the least.

ROMAN NOTATION.

33. In Roman Notation seven Capital letters were used to express numbers. The letters were

I	standing	for	the number	1
V	"	"	"	5
X	"	"	"	10
L	"	"	"	50
C	"	"	"	100
D	"	"	"	500
M	"	"	"	1000

All other numbers being expressed by combining these letters in different ways. Hence Roman Notation is the expression of numbers by letters.

NOTE TO TEACHER.—As the writing of numbers by Roman Notation requires a more extended idea of Addition and Subtraction than the pupil has yet obtained, it may be omitted until after these operations have been learned.

34. They are combined with the following results:

1. When any letter is repeated its value is repeated. Thus, X stands for 10, and XXX stands for 30; C stands for 100, CC stands for 200.
2. When a letter of less value follows one of greater value, its own value must be added to that of the greater.

   Thus, V = 5, VIII = 8, C = 100, CIII = 103, LX = 60.
3. When a letter of less value comes before one of greater value it takes away its value from that of the greater.

   Thus, X = 10, IX = 9, L = 50, XL = 40.
4. When a letter of less value stands between two of greater value the less must be taken from the one that follows it, and the remainder must be added to the one that precedes it.

   Thus XIX = 19, CXL = 140, CXC = 190.
5. A bar over a letter or letters makes it as many thousands as there are units in the letter or letters.

   Thus $\overline{V}$ = 5000, $\overline{L}$ = 50,000, $\overline{C}$ = 100,000, $\overline{IX}$ = 9,000.

☞ A letter is not usually repeated more than three times. Thus 400 would be written CD rather than CCCC.

35. These methods will be found quite sufficient to form all ordinary numbers, and although this Notation could be used in business and other calculations, the process would be very tedious, and for this reason is not used. The characters are chiefly employed to mark the hours on clocks and watches, to number volumes and chapters of books, to indicate the values of coins, bank-bills, etc.

TABLE OF ROMAN NOTATION

I	One.	X	Ten.
II	Two.	XI	Eleven.
III	Three.	XIV	Fourteen.
IV	Four.	XV	Fifteen.
V	Five.	XVI	Sixteen.
VI	Six.	XVIII	Eighteen.
IX	Nine.	XIX	Nineteen.

XX	Twenty.	CD	Four hundred.
XXI	Twenty-one.	D	Five hundred.
XXX	Thirty.	DC	Six hundred.
XL	Forty.	DCC	Seven hundred.
L	Fifty.	DCCC	Eight hundred.
LX	Sixty.	CM	Nine hundred.
XC	Ninety.	M	One thousand.
C	One hundred.	MM	Two thousand.

$\overline{\text{XXV}}$	25000	$\overline{\text{V}}$	5000	$\overline{\text{DL}}$CXL	550140
$\overline{\text{CXX}}$	120000	$\overline{\text{M}}$	1000000	$\overline{\text{M}}$DXC	1000590
$\overline{\text{CL}}$XIV	164000				

EXERCISE 4.

Write the following in Arabic and also in Roman Notation:

1. Thirteen.
2. Seventeen.
3. Nineteen.
4. Twenty-six.
5. Thirty-eight.
6. Forty-four.
7. Ninety-seven.
8. One hundred and fifty.
9. Two hundred and eighty.
10. Seven hundred and thirty-eight.
11. Eight hundred and forty-four.
12. Twelve hundred,
13. Eighty-seven.
14. Six thousand.
15. Fifteen hundred.
16. Eleven thousand.
17. Eight hundred and eighty-eight.
18. Seven thousand five hundred and ninety-two.
19. Four thousand seven hundred and eleven.
20. Fifty-two.
21. Thirty-nine.
22. Forty-three.
23. Sixty-seven.
24. Ninety-one.
25. One thousand eight hundred and eighty-one.
26. Twenty-seven.
27. Forty-nine.
28. Seventy-three.
29. Sixty-eight.
30. Eighty-four.
31. Ninety-seven.
32. One hundred and ten.
33. Five hundred and fifty.
34. Seven hundred and forty.
35. Nine hundred and ninety.
36. Sixteen hundred.
37. Fifty thousand and five.
38. Three hundred and eighteen.
39. Seven hundred and ninety-six.
40. One thousand and ninety-six.
41. Twenty-five thousand.
42. Fifty-nine thousand three hundred.
43. Eighty-seven thousand and forty.

NUMERATION.

36. Numeration is the art of expressing in words those numbers that may be given in figures or letters.

37. If the pupil clearly understands the method of expressing numbers in Notation, there will not be much trouble in reading any number that may be set down in figures. To read large numbers more easily, the figures are separated by commas into periods or groups, commencing from the right hand; and the method that is nearly always adopted is that in which each period or group consists of **three** figures.

38. The first or right hand period contains units, tens, and hundreds, and is called the **period of units**; the second period contains thousands, ten-thousands, and hundred-thousands, and is called the **period of thousands**; the third period contains millions, ten-millions, and hundred-millions, and is called the **period of millions**; and so on for the others. This can be more clearly seen from the following table:—

(Hundred-Trillions. Ten-Trillions. Trillions.)	(Hundred-Billions. Ten-Billions. Billions.)	(Hundred-Millions. Ten-Millions. Millions.)	(Hundred-Thousands. Ten-Thousands. Thousands.)	(Hundreds. Tens. Units.)
8 9 6	5 4 3	2 7 5	4 5 6	1 4 5
5th Period. TRILLIONS.	4th Period. BILLIONS.	3rd Period. MILLIONS.	2nd Period. THOUSANDS.	1st Period UNITS.

The value of the figures in the above table, expressed in words, is eight hundred and ninety-six **trillion**, five hundred and forty-three **billion**, two hundred and seventy-five **million**, four hundred and fifty-six **thousand**, one hundred and forty-five. This table may be extended to any number of orders. The periods after **trillions** are, in their order, **quadrillions, quintillions, sextillions, septillions, octillions, and nonillions.**

39. Hence we have the following rule for reading large numbers easily:

RULE FOR NUMERATION.

Point off the number into periods of three figures each, beginning at the right hand; then begin at the left hand, and read the figures of each period separately, adding the name of each period except the units' period.

Ex. 1.—Read 261034.

First point off by commas, thus: 261,034. The number will then read

261 thousand, and 34.

Ex. 2.—Read 4604792816.

Point off thus; 4,604,792,816, and read:

4 billion, 604 million, 792 thousand, 816.

EXERCISE 5.

(*a*) Point off, read and write:

1. 116234.	8. 141120.	15. 7640.
2. 65231.	9. 101207.	16. 800900.
3. 20703.	10. 68978.	17. 2568242.
4. 71005.	11. 72020.	18. 1008003.
5. 3104.	12. 80001.	19. 212375647.
6. 48000.	13. 857000.	20. 609003588.
7. 60029.	14. 91029.	21. 897856846.

(*b*) Write in figures and read:

1. Nine in the 1st period.
2. Two hundred in the 1st period.
3. Sixty in the 2nd period, two in the 1st.
4. Seven hundred in the 3rd period.

(*b*) Write in figures and read :

5. Two hundred and thirty in the 3rd period, sixty in the 1st.
6. Eighty-one in the 4th period, five hundred and one in the 3rd, seven in the 2nd, twelve in the 1st.
7. Thirty in the 5th period, six hundred and three in the 1st.
8. Seven hundred in the 5th period, eighty in the 4th.
9. Eight in the 4th period, seven in the 3rd, fourteen in the 2nd, and ten in the 1st.
10. Fifteen in the 6th period, eighteen in the 4th, two hundred and seven in the 3rd, and eighty-one in the 1st.

(*c*) Point off, read and write:

1. 60701892.
2. 50607801.
3. 600000.
4. 49000000.
5. 593006070500.
6. 190190001900.
7. 163194568.
8. 3050050183.
9. 5000204.
10. 594900.
11. 12000012.
12. 2007980134.

(*d*) Write in figures:

1. Eighteen in the 2nd period.
2. Two in the 3rd period, sixty in the 2nd, one hundred and fifty-three in the 1st.
3. Sixty in each of the 4th, 3rd, 2nd and 1st periods.
4. 60 million, 200 thousand, 500.
5. 402 billion, 348 million, 213 thousand, 20.
6. 78 trillion, 640 billion, 0 million, 6 thousand, 16.
7. 6 billion, 542 million, 25.
8. Six billion, five hundred and forty-two million, twenty-five.
9. Four hundred and two billion, three hundred and forty-eight million, two hundred and thirteen thousand and twenty.

(*d*) Write in figures:

10. Five million, eight thousand, nine hundred and forty-nine.
11. Two hundred million, three hundred thousand, eight hundred.
12. Twenty-nine billion, five hundred and ninety-nine million, six hundred and one.
13. Four trillion, five hundred and fifty-eight million, two hundred and forty-thousand and seventy.
14. Thirty-two billion, one million, three hundred and forty-three thousand, four hundred and four.
15. Five hundred and fifty-five million, seven hundred and seventy-seven thousand, six hundred and sixty-nine.
16. Eight hundred and six billion, seventy million, five thousand, two hundred and six.

(*e*) Express in Arabic Notation, and also in words:

1. XIX.
2. XXI.
3. X.
4. XLV.
5. LXV.
6. LXIV.
7. LXXIX.
8. LXXXV.
9. CX.
10. CXIX.
11. C.
12. CXIV.
13. CLX.
14. CXC.
15. CCLX.
16. CCXC.
17. DCXXIX.
18. DCCCXI.
19. CML.
20. MCCLIX.
21. LXXVI.
22. MDCCCLXIX.
23. $\overline{V}$CXCIII.
24. XVII.
25. MMCCXXII
26. MMMD.
27. CDL.
28. XLVIII.
29. DXXXVI.
30. MDCCXCIV.
31. $\overline{X}$CXVI.
32. CCCLXXXI.
33. $\overline{L}$CMXCIX.
34. $\overline{M}$MDCXII.
35. $\overline{V}$CDLXX.
36. CCLXV.
37. MMMDCXXVII
38. xix.
39. liv.
40. cdii.
41. dxxxvi.

(*e*) Express in Arabic Notation, and also in words:

42. lxxxv.
43. xviii.
44. lxxvii.
45. LXVII.
46. CLXIV.
47. CXXXV.
48. CXLIX.
49. MXIX.
50. DCLIII.
51. $\bar{C}$XCIX.
52. $\bar{V}$DLIX.
53. $\bar{D}$LX.
54. $\bar{X}\bar{X}\bar{X}\bar{I}$D.
55. $\bar{L}\bar{I}\bar{X}$CCCXLIV.
56. $\bar{X}\bar{V}$DCCXLIX.
57. $\bar{M}$MMMXC.
58. $\bar{V}$MDCCXLIX.
59. $\bar{M}\bar{D}$XXV$\bar{C}$DLXXIX.
60. MDCCCLXXXII.

(If more examples are required for practice, those found in the subsequent Exercises will answer the purpose.)

ADDITION.

40. The Addition of two or more numbers is the method of finding how many units are in all the numbers if taken together.

The number of units found is called the **sum** of the given numbers.

Thus the sum of 4 and 5 is 9, for in 4 there are four units and in 5 there are five units, and if we count five more units after we get four we obtain 9 units.

41. This is often written $4 + 5 = 9$, the sign $+$ meaning that the numbers on each side of it are to be added together. It is called **Plus.**

42. The sign $=$ means that the expressions on each side of it are equal, or of the same value. It is read **Equal to.**

Ex. $3 + 6 = 9$ would be read **three plus six equal to nine**, and it means that the sum of 3 and 6 is nine.

43. The following table should be committed to memory by the pupil, who should at the same time be quite sure, by counting, that the sums are correct:

ADDITION TABLE.

1 and			2 and			3 and			4 and		
1	are	2	1	are	3	1	are	4	1	are	5
2	"	3	2	"	4	2	"	5	2	"	6
3	"	4	3	"	5	3	"	6	3	"	7
4	"	5	4	"	6	4	"	7	4	"	8
5	"	6	5	"	7	5	"	8	5	"	9
6	"	7	6	"	8	6	"	9	6	"	10
7	"	8	7	"	9	7	"	10	7	"	11
8	"	9	8	"	10	8	"	11	8	"	12
9	"	10	9	"	11	9	"	12	9	"	13
10	"	11	10	"	12	10	"	13	10	"	14

5 and			6 and			7 and			8 and		
1	are	6	1	are	7	1	are	8	1	are	9
2	"	7	2	"	8	2	"	9	2	"	10
3	"	8	3	"	9	3	"	10	3	"	11
4	"	9	4	"	10	4	"	11	4	"	12
5	"	10	5	"	11	5	"	12	5	"	13
6	"	11	6	"	12	6	"	13	6	"	14
7	"	12	7	"	13	7	"	14	7	"	15
8	"	13	8	"	14	8	"	15	8	"	16
9	"	14	9	"	15	9	"	16	9	"	17
10	"	15	10	"	16	10	"	17	10	"	18

9 and			10 and			11 and			12 and		
1	are	10	1	are	11	1	are	12	1	are	13
2	"	11	2	"	12	2	"	13	2	"	14
3	"	12	3	"	13	3	"	14	3	"	15
4	"	13	4	"	14	4	"	15	4	"	16
5	"	14	5	"	15	5	"	16	5	"	17
6	"	15	6	"	16	6	"	17	6	"	18
7	"	16	7	"	17	7	"	18	7	"	19
8	"	17	8	"	18	8	"	19	8	"	20
9	"	18	9	"	19	9	"	20	9	"	21
10	"	19	10	"	20	10	"	21	10	"	22

44. The following method will make the pupil quick at adding numbers, and should be practised by recitation as well as on the slate or paper:

Ex. 1.—Add by 2's from 1 to 19.

Result.—1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

Reason.—1 and 2 make 3, and 2 make 5, and 2 make 7, and so on.

Ex. 2.—Add by 2's and 3's alternately, or one after the other, from 1 to 31.

Result.—1, 3, 6, 8, 11, 13, 16, 18, 21, 23, 26, 28, 31.

Reason.—1 and 2 make 3, and 3 make 6, and 2 make 8, and 3 make 11, and so on.

EXERCISE 6.

☞ These questions should be solved mentally.

Add:

By 2's from 4 to 50.
By 3's from 1 to 43.
By 3's from 6 to 51.
By 4's from 1 to 53.
By 4's from 5 to 45.

By 5's from 1 to 61.
By 5's from 7 to 82.
By 6's from 0 to 72.
By 6's from 2 to 80.
By 6's from 10 to 88.

By 2's and 4's alternately from 1 to 49.

By 3's and 4's alternately from 0 to 56.

By 2's and 5's alternately from 4 to 60.

By 3's and 5's alternately from 7 to 63.

Begin with 1 and add 2 successively till the sum equals 101.

Begin with 2 and add 2 successively till the sum equals 100.

Begin with 2 and add 3 successively till the sum equals 101.

Begin with 3 and add 3 successively till the sum equals 102.

Begin with 3 and add 4 successively till the sum equals 103.

Begin with 4 and add 5 successively till the sum equals 104.

Begin with 5 and add 7 successively till the sum equals 103.

Begin with 6 and add 8 successively till the sum equals 102.

Begin with 9 and add 9 successively till the sum equals 108.

4+2+3+7+1+5+9+6+3= what?

8+3+5+9+2+0+4+7+8= what?

6+5+2+1+9+7+6+8+4= what?

9+8+6+7+4+1+5+3+8= what?

7+8+9+0+7+8+9+0+7= what?

8+9+5+8+9+7+0+9+8+2+6+1+4+3= what?

Add:

By 7's from 1 to 71.	By 10's from 0 to 120.
By 7's from 3 to 87.	By 10's from 13 to 153.
By 8's from 0 to 96.	By 11's from 1 to 100.
By 8's from 6 to 102.	By 11's from 4 to 92.
By 9's from 2 to 92.	By 12's from 0 to 144.
By 9's from 10 to 109.	By 12's from 3 to 135.

Add rapidly the following:

4, 6, 5, 3, and 7.	13, 5, 6, 10, and 3.
6, 4, 8, 2, and 5.	12, 10, 2, 0, and 9.
10, 9, 5, 3, and 6.	27, 10, 3, 8, and 7.
10, 3, 7, 9, and 8.	36, 12, 7, 4, and 10.
14, 5, 3, 6, and 10.	11, 12, 10, 9, and 8.

Add the numbers,

2, 3, 4, 2, 3, 4, 2, 3, 4, etc., till the sum = 63.

3, 4, 5, 3, 4, 5, 3, 4, 5, etc., till the sum = 84.

2, 4, 6, 2, 4, 6, 2, 4, 6, etc., till the sum = 96.

Add alternately,

5, 6, 5, 6, 5, 6, 5, 6, 5, 6, etc., till the sum = 88.
6, 4, 6, 4, 6, 4, 6, 4, 6, 4, etc., till the sum = 100.
7, 5, 7, 5, 7, 5, 7, 5, 7, 5, etc., till the sum = 120.
8, 9, 8, 9, 8, 9, 8, 9, 8, 9, etc., till the sum = 119.

---

45. All numbers may be added together by means of the preceding table, and we will first add numbers when the sum of each order in them is less than 10.

46. **To add numbers together, they must be of the same kind.**

Ex.—5 peaches and 4 apples, if added together, make neither 9 peaches nor 9 apples. In the same way, 5 tens and 4 units make neither 9 tens nor 9 units.

47. Hence, when we add, we should always place figures of the **same kind** in the same column, that is, **units** under **units, tens** under **tens, hundreds** under **hundreds**, etc.

48. We thus have the following

RULE FOR ADDITION.

1. *Write the numbers to be added, placing figures of the same kind in the same column, that is, units under units, tens under tens, etc., and draw a line beneath.*

2. *Begin at the right, and add the figures in each column from bottom to top.*

3. *Place under each column the result obtained by adding its figures.*

49. To test the result, proceed in the same way, adding from top to bottom. The two sums should be the same, if the work be correct.

Ex. 1.—Find the sum of 422, 342, and 134.

hundreds	tens	units
4	2	2
3	4	2
1	3	4
8	9	8

We write the numbers, placing **units** under **units**, **tens** under **tens**, **hundreds** under **hundreds**. We begin at the units' column and say: 4 units and 2 units are 6 units, and 2 units are 8 units, and we write the 8 under the units' column. Next, 3 tens and 4 tens are 7 tens, and 2 tens are 9 tens, and we write the 9 under the tens' column. Next, 1 hundred and 3 hundreds are 4 hundreds, and 4 hundreds are 8 hundreds, and we write the 8 under the hundreds' column. Hence, the entire sum is 8 **hundreds** 9 **tens** and 8 **units**, or 898.

In **practice** we shorten the work in this way: 4 and 2 are 6, and 2 are 8; 3 and 4 are 7, and 2 are 9; 1 and 3 are 4, and 4 are 8; making 898.

Ex. 2.—Add 3214, 2312, and 3453.

3214	Here 3 and 2 are 5, and 4 are 9, which we place under units' column. Next, 5 and 1 are 6, and 1 are 7, which we place under tens' column. Next, 4 and 3 are 7, and 2 are 9, which we write under hundreds' column. Lastly 3 and 2 are 5, and 3 are 8, which we set under thousands' column, making the sum 8979.
2312	
3453	
8979	

EXERCISE 7.

☞ The first thirty of the following questions should be worked mentally.

(*a*) 1. A farmer harvested five loads of hay one day and six loads another day; how many loads did he harvest in the two days?

2. Henry's father gave him seven cents, and his brother gave him five: how many cents did he get in all?
3. John worked six days one week and six days the next week: how many days did he work in the two weeks?
4. I have six cherries in one hand and eight cherries in the other: how many have I in both hands?
5. Thomas has seven apples in one basket and nine in another: how many has he in both?
6. There are seven sheep in one pen and seven in another: how many sheep are in the two pens?
7. How many are seven dollars, four dollars, and two dollars?
8. I had nine trees in the garden, and I set out five more: how many trees have I now in the garden?
9. Cora bought some paper for eight cents and some pens for eight cents: how much did she spend?
10. James had eight plums, Joseph had nine, and John had four: how many had they in all?
11. There were ten wild ducks flying, when four more joined them: how many ducks were then in the flock?
12. How many are eight cents, six cents, and five cents?
13. There are seven books on one desk and six on another: how many books are on the two desks?
14. Joseph had three cents, his aunt gave him five, and his brother gave him eight: how many had he then?
15. One hen had four chickens, and another had nine: how many chickens were there altogether?
16. Herbert had four apples, his brother gave him three, and his sister two: how many did he then have?
17. One word contains ten letters, and another seven: how many letters are there in the two words?
18. Mary had nine books, and her mother gave her three more: how many had she then?
19. A man gave nine dollars for a plough, eight dollars for a rake, and six for a harrow: how much did he give for all?

20. How many are nine dollars, three dollars, and four dollars?

21. David gave seven cents for apples, eleven cents for pears, and eight cents for peaches; how many cents did he spend?

22. How many are six and three and five?

23. A boy bought a pencil for ten cents and some pens for five; what did both cost?

24. How many are eight birds, seven birds, and two birds?

25. William spent nine cents for pears and eight for plums; how many cents did he spend altogether?

26. In one window there are nine panes of glass, in another six; how many are there in the two?

27. How many are eight and five and three?

28. Susan had eleven pears; her father gave her five, and her mother three; how many had she then?

29. How many are seven and five and six?

30. Jane paid six cents for silk, seven cents for a spool of thread, nine cents for pins, and four cents for tape; how much did she pay for all?

31. A man owns 4 farms; the first contains 1143 acres, the second 2320 acres, the third 3425 acres, and the fourth 2010. How many acres does he own?

(*b*) Add together

(1)	(2)	(3)	(4)	(5)	(6)
23	43	27	36	72	123
62	36	31	22	17	241
(7)	(8)	(9)	(10)	(11)	(12)
181	5431	7654	5346	6135	4523
712	2364	1235	2453	3844	2236
(13)	(14)	(15)	(16)	(17)	(18)
7840	2253	1122	1216	3701	21020
2105	4314	2203	2701	1293	34917
33	2432	3322	1082	2005	22032

(*b*) Add together:

(19)	(20)	(21)	(22)	(23)	24)
313291	133072	3093124	202020	9334567	1212021
201306	101	2101003	333222	40101	2301304
211002	3303	3003251	262626	623311	33330
123100	12322	2020	102101	2020	201000

50. We now come to the case, in which the sum in any column is equal to 10, or more than 10.

Ex.—Add together 378, 691, and 421.

$$\begin{array}{r} 378 \\ 691 \\ 421 \\ \hline 1490 \end{array}$$

Writing **units** under units, **tens** under **tens**, etc., as before, we say: 1 unit and 1 unit are 2 units, and 8 units are 10 units, which are equal to 1 **ten** and **0 units.** Set down the 0 units under the units' column and take the 1 ten to the next, or tens' column. Then, 1 ten (which we carry) and 2 tens are 3 tens, and 9 tens are 12 tens, and 7 tens are 19 tens, which are equal to 1 **hundred** and 9 **tens.** Set the 9 **tens** under the tens' column and carry the 1 **hundred** to the next or the hundreds' column. Then, 1 hundred (which we carry) and 4 hundreds are 5 hundreds, and 6 hundreds are 11 hundreds, and 3 hundreds are 14 hundreds, which are equal to 1 **thousand** and 4 **hundreds.** Set down the 4 hundreds under its own column, and as there is no thousands' column, we must simply place the 1 thousand under the place where the thousands' column would be. Thus we have the answer, 1490.

In short:

1 and 1 are 2, and 8 are 10. Set down the 0 and carry the 1. Then, 1 and 2 are 3, and 9 are 12, and 7 are 19. Set down the 9 and carry the 1. Again, 1 and 4 are 5, and 6 are 11, and 3 are 14. Set down the 4 and carry the 1 to the next place to the left, or, in other words, set down the 14.

51. The work might be written thus:

3	hundreds	+	7	tens	+	8	units
6	"	+	9	"	+	1	"
4	"	+	2	"	+	1	"
13	hundreds	+	18	tens	+	10	units

Now 10 units	=					1	ten	+0	units
18 tens	=		1	hundred	+8	tens	+0	units	
13 hundreds	=	1	thousand	+3	hundreds	+0	tens	+0	units
		1	thousand	+4	hundreds	+9	tens	+0	units

This is exactly the same result as we had before.

52. We thus obtain the following complete

RULE FOR ADDITION.

1. *Write the numbers to be added, placing figures of the same kind in the same column.*
2. *Begin at the right hand and add each column separately. If the amount of any column be less than* 10, *place it under the column added; but if the amount be* 10 *or more, place the right-hand figure of the amount under the column added, and carry the left-hand figure or figures to the next column.*
3. *Proceed in the same way through all the columns, and set down the whole amount of the last column.*

53. As in the former case, the best way of proving the correctness of the result is by adding from the top line downwards.

54. In adding numbers, the pupil should be always taught to add without repeating the sum each time a new figure is added.

Ex.—Add together 869, 4931, 2687, 1072.

	869
	4931
	2687
	1072
Ans.	9559

Beginning at the right, we say 2 and 7 are 9, and 1 are 10, and 9 are 19, instead of saying, 2 and 7 are 9, 9 and 1 are 10, 10 and 9 are 19.

After placing down the 9 units we carry the 1 ten, and begin to add the second column in the same way, saying, 1 and 7 are 8, and 8 are 16, and 3 are 19, and 6 are 25, and so on for all the columns.

EXERCISE 8.

1. Add together:

(1)	(2)	(3)	(4)	(5)
756	425	127	106	6204
895	143	341	341	2413
784	231	210	121	1231

(6)	(7)	(8)	(9)	(10)
7856	276	748	4681	36487
8943	483	249	7362	10462
6789	874	838	8428	38420
4584	965	749	1697	79549

(11)	(12)	(13)	(14)	(15)
417	251	4376	5438	59763
819	432	8231	7846	47
234	846	2343	829	8623
846	735	5678	9731	437
721	897	658	96	24675

(16)	(17)	(18)	(19)	(20)
749831	12734	3786	8763	1379463
8632	63741	97643	48694	207839
54317	32347	278	7687	999
432613	87698	89784	24	7638
48	37	3264	89643	72109
460	794	1640	159	367294

2. Find the sum of $79+863+900832+8632+473423+7+637$.

3. Find the sum of $12+36701+10+9+8764+99910$.

4. Find the sum of 789632+4+67+879002+876+970.
5. Find the sum of 98632+76398+832+97+10029+7384.
6. Find the sum of 1324+4354653+12+876+97843+68473.
7. Find the sum of 98673 + 10370634 + 97 + 964732 + 84944849+98+9.
8. What is the sum of 824786+79+876345+37341+632?
9. What is the sum of 34688+637+423+98976+38?
10. Add 181+24+897156+881+71512.
11. What is the sum of 82+873+93+9+824683+1000201?
12. Find the sum of 3247+864302+84+96703+10001003.
13. If 83976 be added to 74932, then their sum to 389727, and this sum be added to the sum of 83791 and 932016, what will be the total sum?
14. Add together the answers in questions 3, 7, 9, 10, 12.
15. Add together:

(1)	(2)	(3)	(4)
348037	460375	963172	849652
272465	841681	300725	361728
530634	239724	463248	412381
109871	763256	721003	635403
693036	437891	387356	872545
764543	825432	241653	406223
233638	285678	603280	294867
428432	310720	532176	811236
389763	403521	278321	576037
210045	687489	829248	213744
760806	324061	171320	764368
636215	530724	206782	305216
253734	623452	461027	436720
251600	487638	589203	823284
575453	290731	248639	217436
807720	803256	730461	592301
930046	731463	672398	243762
174173	379574	246175	739445
626245	823156	928340	429374
342734	928348	731629	684569

16. On my farm are 1640 oak, 748 ash, 639 beech, 184 birch, 597 fir, 48 poplar, 186 apple, and 247 pear trees: how many trees have I in all?

17. In one year, a farmer made by his horses $364, by his cows $785, by his sheep $1064, by his pigs $184, by his corn $304, by his hay $97, and by his oats $46: what did he make in all?

18. One shepherd has 327 sheep, another has 25 more than he: how many have both?

19. By selling a farm for $6478 the owner lost $2734: what did it cost at first?

20. James was born in 1856: in what year will he be 48 years old?

21. The first of three numbers is 398, the second is 67 more than the first, and the third 825 more than the second: find the sum of the three.

22. A grocer lost $957, and had left $5635: how much would he have had, if he had gained $957 instead of having lost that sum?

23. Commencing with 324, what is the sum of all the numbers below 335?

24. The population of a village was 1527, the next year it gained 421, the second year 923, the third year 845, the fourth 1760, the fifth 1099: what was the population at the end of the fifth year?

25. A man left his estate to his wife, two sons, and three daughters. His wife received $9527, each son $5726, and each daughter $3784: what was the value of the whole estate?

26. A. owns a farm worth $8920, three horses worth $150 each, a pair of oxen worth $98, five cows worth $25 apiece, and sheep worth $1832: what is the total value of his property?

27. Washington was born in 1732, and Napoleon 36 years later; Napoleon died at the age of 53: in what year did he die?

28. The population of a certain city was 23000 in the year 1845; in the next five years it gained 5630; in the next five, 8763; in the next 16420; in the next, 22109: how many inhabitants had it at the end of the latter time?

29. An army consists of 6450 cavalry, 27846 artillery, and 270874 more infantry than both cavalry and artillery: how many men were there in the army?

30. A. has $5786; B., $6724; C., $10536; D. as much as A. and C.; E. as much as A., B., and D.; F. as much as all the rest: how much have they in all?

31. A. has $84; B. has $23 more than A.; C. has as much as A. and B.; and D. as much as A., B., and C. together: how many dollars have they in all?

32. A. deposits his money in five banks; in the first he has $897, in the second $673, in the third as much as in the first and second, in the fourth as much as in the second and third, and in the fifth as much as in all the others: how much money has he deposited in all?

33. Three men enter into partnership; the first man puts $3845, the second $2375, and the third puts in $585 more than the sums put in by the other two: what sum did they all put in?

34. A man builds seven houses; for the first he receives $647, for the second $799, for the third $949, for the fourth $1467, for the fifth $1986, for the sixth as much as for the first and fourth, and for the seventh as much as for the third and fifth: how much does he receive for building the sixth and seventh respectively, and how much for building them all?

35. Five persons deposited money in the same bank; the first deposited $5987, the second $12980, the third $65973, the fourth $37345, and the fifth as much as the first and second together: how many dollars did they all deposit?

36. The first of four numbers is 3125, the second is greater than the first by 5108, the third is equal to the sum of

the first and second, and the fourth is equal to the sum of the third and first: what is the sum of the four numbers?

37. The ship *Orient* sailed from Marseilles to Buenos Ayres, distant 6375 miles, thence to Valparaiso 2764 miles, thence to San Francisco 6346 miles, thence to the Sandwich Islands 2152 miles, thence to Melbourne 5588 miles, thence to Yokohama 5434 miles, thence to Calcutta 5115 miles, thence to Bombay 2257 miles, thence to Suez 2006 miles, and thence back to Marseilles 1314 miles: what was the entire distance sailed?

38. Find the sum of four hundred and three; 5025; sixty thousand and seventy; eighty-seven thousand; 2090; one hundred and three.

39. Find the sum of 2050; three hundred and seventy thousand and two hundred; four million and five; two million, ninety thousand, seven hundred and eighty; one hundred thousand and seventy; 98002; seven million, nine thousand and one; 70070.

40. Find the sum of two hundred thousand, two hundred; three hundred million, six thousand and thirty; seventy million, seventy thousand and seventy; nine hundred and four million, nine thousand and forty; eighty thousand; ninety million, nine thousand; six hundred thousand and sixty; five thousand, seven hundred; four million, twenty thousand, eight hundred and twenty.

41. Find the sum of all the different numbers you can make by using all the figures:

(1) 3, 0, 2, 0.
(2) 3, 8, 9.
(3) 7, 9, 0, 0.

42. Add together all the different numbers of five figures, each number beginning with 375, and ending with 9.

43. Lily has 17 roses; Laura has 11 more than Lily, Charles has 18 dahlias more than Laura has roses, and Jennie has 14 dahlias more than Charles; how many dahlias has Jennie?

44. Henry's purse contains 329 cents; Edward's contains 43 more than Henry's, and Henry's contains as many cents as Sarah's, less 94 cents: how many cents does Sarah's purse contain?

45. Mary bought a pencil for which she gave 95 cents; Maude bought one for which she gave 13 cents more than Mary; and Maude's pencil cost as much as Jane's, less 23 cents: what was the cost of Jane's pencil?

46. Henry lends $913 to Thomas, $473 to Samuel, $576 to Theodore, and has $576 left: how many dollars had he at first?

47. A man was 37 years old when his son was born: how old will he be, when his son has reached the age of 59?

48. John throws a ball 30 yards up the road, and another 40 yards down the road: how far must he walk to bring them both back again?

SUBTRACTION.

55. By Addition we find that 7 units and 4 units make 11 units. We will now find what 11 units become when 4 units are taken away. If 1 of the 4 units be taken from 11, the result will be 10; if 1 of the remaining 3 units be taken from 10, there will be 9 units left. Again, take 1 of the remaining 2 units from the 9 and 8 will remain; and, finally, take the 1 remaining unit from 8 and we have 7 units left. Thus we see, that if 4 be taken from 11 there will be 7 left.

56. This process of finding the number of units left after taking a certain number from a greater number is called **Subtraction.**

57. The greater number, as the 11 in the above example, is called the **Minuend,** and the lesser, as the 4 above, is called the **Subtrahend.** That which is left, as the 7 above, is called the **Difference, Remainder,** or **Excess.**

58. This operation is expressed by placing a sign − between the two numbers. The sign − is called **Minus.**

Thus, 10 − 3 = 7.

This is read:—**Ten minus** three equal to 7, and means that if 3 units be taken from 10 units 7 units will be left. Here 10 is the **Minuend,** 3 is the **Subtrahend,** and 7 is the **Remainder.**

59. The following table should be perfectly understood and remembered by the pupil:

SUBTRACTION TABLE.

1 from			2 from			3 from			4 from			5 from		
1	leaves	0	2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
2	"	1	3	"	1	4	"	1	5	"	1	6	"	1
3	"	2	4	"	2	5	"	2	6	"	2	7	"	2
4	"	3	5	"	3	6	"	3	7	"	3	8	"	3
5	"	4	6	"	4	7	"	4	8	"	4	9	"	4
6	"	5	7	"	5	8	"	5	9	"	5	10	"	5
7	"	6	8	"	6	9	"	6	10	"	6	11	"	6
8	"	7	9	"	7	10	"	7	11	"	7	12	"	7
9	"	8	10	"	8	11	"	8	12	"	8	13	"	8
10	"	9	11	"	9	12	"	9	13	"	9	14	"	9
11	"	10	12	"	10	13	"	10	14	"	10	15	"	10

6 from			7 from			8 from			9 from			10 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0	10	leaves	0
7	"	1	8	"	1	9	"	1	10	"	1	11	"	1
8	"	2	9	"	2	10	"	2	11	"	2	12	"	2
9	"	3	10	"	3	11	"	3	12	"	3	13	"	3
10	"	4	11	"	4	12	"	4	13	"	4	14	"	4
11	"	5	12	"	5	13	"	5	14	"	5	15	"	5
12	"	6	13	"	6	14	"	6	15	"	6	16	"	6
13	"	7	14	"	7	15	"	7	16	"	7	17	"	7
14	"	8	15	"	8	16	"	8	17	"	8	18	"	8
15	"	9	16	"	9	17	"	9	18	"	9	19	"	9
16	"	10	17	"	10	18	"	10	19	"	10	20	"	10

NOTE TO TEACHER.—Drill the pupils thoroughly in this table by asking questions in every possible form.

60. The following method will teach pupils to be quick at subtracting numbers, and should be practised aloud as well as on the slate or paper:

Ex. 1.—Subtract by 2's from 19 to 1.

Result.—19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

Reason.—2 from 19 leaves 17; 2 from 17 leaves 15, etc.

Ex. 2.—Subtract by 3's and 2's, one after the other, from 31 to 1.

Result.—31, 28, 26, 23, 21, 18, 16, 13, 11, 8, 6, 3, 1.

Reason.—3 from 31 leaves 28; 2 from 28 leaves 26; 3 from 26 leaves 23; 2 from 23 leaves 21, etc.

☞ **Compare these results with Art. 44, and notice that Subtraction is exactly the converse or opposite of Addition.**

EXERCISE 9.

☞ These questions should be solved mentally.

1. If John is 15 years old and George is 6, what is the difference in their ages?
2. How many are 16 cents − 7 cents?
3. How many are 18 dollars − 5 dollars?
4. How many are 14 − 6? 16 − 4? 12 − 5?
5. How many are 18 − 8? 20 − 6? 21 − 4?
6. Five balls taken from 11 balls leave how many?
7. Six cents from 20 cents leave how many?
8. How many are 7 − 5? 17 − 5? 27 − 5?
9. How many are 9 − 6? 19 − 6? 29 − 6?
10. What number added to 8 will make 12?
11. What number and 9 make 13? 14? 15? 16?
12. Subtract by 2's from 24 to 0.

In the same manner, subtract

13. By 2's from 25 to 1.
14. By 2's from 31 to 3.
15. By 3's from 30 to 0.
16. By 3's from 37 to 1.
17. By 3's from 40 to 4.
18. By 4's from 44 to 0.
19. By 4's from 41 to 1.
20. By 4's from 51 to 3.
21. By 5's from 60 to 0.
22. By 5's from 63 to 3.
23. By 6's from 66 to 0.
24. By 6's from 65 to 5.
25. Count by 4's from 2 to 58, and back from 58 to 2.
26. Count by 5's from 1 to 61, and back to 1.
27. Count by 6's from 3 to 69, and back to 3.
28. Count by 4's from 5 to 53, and back to 5.
29. Count by 6's from 7 to 67, and back to 7.
30. What is the difference between nine dollars and fifteen dollars?
31. A man earned fifteen dollars one week, and spent seven dollars: how much had he left?
32. A man has two sons, one fourteen years old, the other eight: what is the difference between their ages?
33. Take nine yards from thirteen yards: how many yards remain?
34. A man had sixteen dollars: he spent eleven dollars of it for a dictionary, and the remainder for paper: what did the paper cost?
35. A woman carried eighteen dozen eggs to market; she broke seven dozen, and sold the rest: how many did she sell?
36. William bought sixteen marbles, and he gave his brother seven of them: how many did he keep?
37. Mary had fifteen examples to work out: after finishing nine, how many had she then to do?
38. Take seven books from thirteen books: and how many will remain?
39. A merchant had nineteen barrels of flour; he sold twelve barrels, and kept the rest: how many did he keep?

40. George had seventeen cents; he lost nine cents, and spent the rest for ink: what did the ink cost?

41. There were fourteen books on one shelf of a book-case, and eight on another: how many more books were there on one shelf than on the other?

42. Take eight cherries from fifteen cherries: and how many will remain?

43. Mary is sixteen years old, and Jane seven: how much older is Mary than Jane?

44. A man sold a cart for eighteen dollars; he received for it a barrel of flour worth nine dollars, and the rest in money: how much money did he receive?

45. James's father gave him ten cents, and his mother gave him nine: after spending eight cents, how much had he left?

46. A boy had thirteen marbles; he bought five more, and afterwards lost ten: how many had he then?

47. John had eight books, his father gave him five, his mother two; he then gave four to his brother; how many had he left?

48. A man had sixteen dollars; he gave away seven dollars and afterwards earned nine: how much had he then?

49. A merchant bought some cloth for nine dollars, and some silk for five dollars; he sold both for sixteen dollars: did he gain or lose by the bargain: if so, how much?

---

61. **To** subtract one number from another they must be of **the same kind.**

Ex.—5 dollars from 8 apples leaves neither 3 dollars nor 3 apples. In the same way, 3 units from 7 thousands leaves neither 4 units nor 4 thousands.

62. Hence when we subtract, we should always write numbers of the same kind in the same column, that is, **units** under **units, tens** under **tens, thousands** under **thousands,** etc.

63. We thus have the following

RULE FOR SUBTRACTION.

1. *Write the numbers to be subtracted, the less under the greater, placing units under units, tens under tens, etc., and draw a line underneath.*

2. *Begin at the right, and subtract each figure from the one above, placing the result under the figure subtracted.*

64. To test the result, add the Subtrahend, or middle number, to the Difference, or lower number, and if the work be correct the sum should be the same as the Minuend, or upper number; or, subtract the lower number from the upper, and the result should be the same as the middle number.

Ex. 1.—Subtract 238 from 749.

	hundreds	tens	units
Minuend	7	4	9
Subtrahend	2	3	8
Difference	5	1	1

We place the numbers, **units** under **units, tens** under **tens,** and **hundreds** under **hundreds.** Then begin at the units' column and say: 8 units from 9 units leaves 1 unit, and we write the 1 under the units' column. Then 3 tens from 4 tens leaves 1 ten, and we put the 1 under the ten's column. Then 2 hundreds from 7 hundreds leaves 5 hundreds, and we place the 5 under the hundreds' column. The answer being 511.

In practice we shorten the work thus: 8 from 9 leaves 1; 3 from 4 leaves 1; 2 from 7 leaves 5; leaving altogether 511.

To prove the correctness of the work, we add 238 to 511 and obtain 749; or we may subtract 511 from 749 and obtain 238.

The reason for this is as follows: 4 from 9 leaves 5; and if we add the 4 back again to the 5, we must obtain the 9 we had at first.

Again, if 4 taken from 9 leaves 5, the pupil will easily see that 5 taken from 9 must leave 4.

Hence the proof of the work.

Ex. 2.—Find the difference between 3065 and 78195.

78195
3065
75130

Here 5 from 5 leaves 0, which is placed under the units' column; 6 from 9 leaves 3 under the tens' column; 0 from 1 leaves 1 under the hundreds' column: 3 from 8 leaves 5 under the thousands' column; 0 from 7 leaves 7 in the ten-thousands' column.

EXERCISE 10.

(1)	(2)	(3)	(4)	(5)	(6)
469	5642	9874	8072	2741	5462
327	4130	3623	3051	1301	1350

(7)	(8)	(9)	(10)	(11)	(12)	(13)
6408	8420	8742	7839	1243	4785	86493
3207	3110	6331	5427	123	1053	34272

(14)	(15)	(16)	(17)	(18)
972897	985094	987657899	99797486	9892976
120341	382040	123456789	89762312	4730834

(19)	(20)	(21)	(22)	(23)	(24)
75853	89487	75659	87392	75285	88456
45213	32315	32417	43181	43151	32142

(25)	(26)	(27)	(28)	(29)	(30)
546875	347985	973856	825944	756345	914756
513213	323415	951231	812512	713125	902314

(31)	(32)	(33)	(34)	(35)	(36)
41876	38789	64187	918764	187137	59123
31023	13321	33123	312311	123013	32122

(37)	(38)	(39)	(40)	(41)	(42
418764	13912	67134	67183	91276	41876
213321	13311	32132	21033	21230	31232

43. A boy had 36 marbles and gave 24 of them to his playmate: how many had he remaining?

44. Joseph caught 295 quail, and John caught 184: how many more did Joseph catch than John?

45. Wilson had $6847, and Joseph $2437: how much more had Wilson than Joseph?

46. Thomas having 447 bushels of potatoes, sold 234 bushels of them to Perry: how many bushels had Thomas remaining?

47. A farmer bought a span of horses for $346, and a yoke of oxen for $135: how much more did he give for the horses than for the oxen?

48. A drover having 1465 sheep, sold 1235 of them: how many had he remaining?

49. A gentleman owns a store worth $4695, and a grist-mill worth $2135: how much more is the store worth than the grist-mill?

50. A gentleman gave for a house and lot $9899, and for a cotton factory $8495: how much more did he give for the one than for the other?

51. A speculator bought some land for $12897, and a tannery for $10444: how much more did the land cost than the tannery?

52. A merchant having 9847 yards of cloth, sold 5844 yards of it: how many yards had he remaining?

53. A drover bought cattle to the amount of 9647 dollars, and sheep to the amount of 5434 dollars: how much more did he give for the cattle than for the sheep?

54. Two men jointly built a mill for 7856 dollars; one furnished 4520 dollars: what did the other furnish?

55. The earnings of a factory for a year were 45689 dollars, and the expenses were 21352 dollars: what were the profits?

56. The gross receipts of a railroad were 357845 dollars, and the running expenses for the same time were 213423 dollars: what were the net earnings?

57. A. has a grist-mill worth 1875 dollars, and a saw-mill worth 1032 dollars: how much more is the one worth than the other?

58. A farmer had 3672 sheep and 2312 lambs: how many more sheep had he than lambs?

59. A man was driving 534 geese to market, and on the way had 21 stolen from him: how many had he remaining?

60. A farmer had 327 bushels of oats, and sold 125 bushels of them: how many bushels had he remaining?

61. A merchant in one year sold 18972 barrels of flour and 7370 barrels of sugar: how many more barrels of flour did he sell than sugar?

62. A ship is valued at 54789 dollars, and its cargo at 40357 dollars: how much more is the ship valued at than the cargo?

63. A gentleman having 577897 dollars, gave to his eldest son 16805 dollars: how much had he remaining?

---

65. In all the former examples, the figures of the larger number were either greater or equal to the corresponding figure in the lesser number. We will now consider those cases in which the figures of the larger number may be less than the corresponding figures of the other.

Ex.—Subtract 695 from 932.

$$932 = 900 + 30 + 2$$

hundreds. tens. units.

$$= 9 + 3 + 2$$

In the same way,

		hundreds.		tens.		units
695	=	6	+	9	+	5

The numbers then stand :

H.		T.		U.
9	+	3	+	2
6	+	9	+	5
2	+	3	+	7

Now, we cannot take 5 from 2 units, hence we increase the latter by 1 ten or 10 units, thus making it 12 units; then 5 units from 12 units leaves 7 units, which we place as usual. Again, since we took 1 of the 3 tens to add to the 2 units, we must have only 2 tens left from which to take the 9 tens, and here, since we cannot take 9 from 2, we must take 1 of the 9 hundreds or 10 tens and add it to the 2 tens, making 12 tens. Then 9 tens from 12 tens leaves 3 tens, which we place as usual. Again, having taken 1 hundred from the 9 hundreds, we can have only 8 hundreds left, and since we can take the 6 hundreds from the 8 hundreds we do so, and place the 2 hundreds as usual.

The work may be shown thus :

	H.	T.	U.	H.	T.	U.	H.	T	U.
932	= 9	+ 3	+ 2	= 9	+ 2	+ 12	= 8	+ 12	+ 12
695	=	.	.	.	.	.	= 6	+ 9	+ 5
237	=	.	.	.	.	.	= 2	+ 3	+ 7

66. By another method, it is usual to add **one**, when necessary, to the figures of the Subtrahend, instead of taking **one** away from the figures of the Minuend.

In the previous example, instead of saying 9 tens from 12 tens leaves 3 tens, we say 10 tens from 13 tens leaves 3 tens, which gives the same result; and instead of saying 6 hundreds from 8 hundreds, we say 7 hundreds from 9 hundreds, so that we only change the figures of the Subtrahend.

67. This system, known as **carrying one**, is done mentally while working a question in Subtraction, and depends

on the fact that 1 unit of any order is equal to 10 units of the next order to the right of it.

68. We thus have the following

RULE FOR SUBTRACTION.

1. *Write the less number under the greater, placing units under units, tens under tens, etc., and begin at the right to subtract.*
2. *Subtract, if possible, each figure in the lower line from the one above it, and set the remainder below.*
3. *If any figure in the lower line is greater than the one above it, add* 10 *to the upper figure before subtracting, and diminish by* 1 *the next left-hand figure in the upper line, and proceed as before.*

PROOF.—*Add the remainder to the subtrahend; the sum should be equal to the minuend.*

EXERCISE 11.

	(1)	(2)	(3)	(4)	(5)
a)	663	976	704	1806	572
	580	531	483	720	259
	(6)	(7)	(8)	(9)	(10)
	7238	7580	52836	400500	4236
	4854	4245	28371	215327	3089
	(11)	(12)	(13)	(14)	(15)
	6170	4360	5491	6180	4192
	5552	3451	4542	2435	1435
	(16)	(17)	(18)	(19)	(20)
	80502	927381	917183	618190	519080
	38672	345432	421354	234221	324121
	(21)	(22)	(23)	(24)	(25)
	924390	705180	527082	816141	423453
	432412	443544	232154	135212	141514
	(26)	(27)	(28)	(29)	(30)
	732250	734271	826041	46095	555555
	241341	241342	434425	28736	123456

(*b*) 1. From 854 take 578.
2. From 1799 take 1732.
3. From 5496 subtract 1492.
4. From 1584 subtract 920.
5. From 5672 subtract 2356.
6. From 74760 subtract 39817.
7. From 8416 subtract 2918.
8. From 30811 subtract 13240.
9. From 27880 subtract 9226.
10. From 35846 subtract 12829.
11. From 75901 subtract 17980.
12. From 37229 subtract 17991.
13. From 100304 subtract 62818.
14. From 1000302 subtract 888772.
15. From 892201 subtract 300998.
16. From 1000000 subtract 333333.

(*c*) Find the value of

1. 758901 − 349806.
2. 329500 − 54650.
3. 720065991 − 12095899.
4. 10000 − 390.
5. 189501 − 188605.
6. 75625 − 24319.
7. 78629000 − 17664508.
8. 1370426019 − 820512055.
9. 97001 − 50077.
10. 76734 − 977.
11. 56400 − 100.
12. 700000 − 99.
13. 5700 − 500.
14. 9777 − 89.
15. 76000 − 1.
16. 90017 − 3.
17. 230469 − 85340.
18. 349130 − 94131.
19. 400500 − 80973.
20. 739745 − 76378.
21. 511839 − 84674.
22. 601813 − 13834.
23. 803460 − 45009
24. 910311 − 87300.
25. 999830 − 99001.
26. 7465676 − 567456.
27. 37823 − 18281.
28. 780023 − 320412.
29. 74603 − 43374.
30. 700 − 2.
31. 8004 − 7008.
32. 830240 − 370428.
33. 83001 − 38994
34. 783124 − 291431.
35. 70800 − 8004.
36. 8467321 − 3478271.
37. 3178632 − 1478371.
38. 11247863 − 4613278.
39. 70001 − 7.
40. 9004100 − 30012.
41. 486321 − 198372.
42. 976321 − 123679.
43. 900000 − 1.
44. 100010110 − 990991.

(*d*) 1. From 65 million take 650 thousand 980.

2. From nine hundred thousand take five hundred and 50.

3. From 12 million 12 hundred take 400 thousand 4 hundred.

4. From 280 million and 11 take 24 million 650 thousand.

5. From twelve hundred and ninety take seventy-five.

6. From 6 hundred million take 500 million 5 hundred.

7. From six hundred and 48 take one hundred and 70.

8. From 460 million and 10 take 920 thousand 750.

9. From 1 million and 20 take 960 thousand.

10. From 756 million 3 thousand take 657 million and 8.

11. From six hundred and 90 take seventy-five.

12. What is the difference between 900000 and 123454?

13. How much larger is 38607 than 3867?

14. How much smaller is 34730 than 38607?

15. How much must be taken from 2483 to leave 391?

16. How much must be added to 2032 to make 2483?

17. From 7630005 take 3270006.

18. The larger of two numbers is 10640 and the less 9535: what is their difference?

EXERCISE 12.

1. Mr. A. had 350 sheep in two lots; in one lot were 175 sheep: how many were in the other?

2. How many years have elapsed since the discovery of America in 1492?

3. A. borrowed from B. $9780, and paid $2176: how much remained due?

4. A. purchased a farm for $10000, and paid thereon $4790 how much remained due?

5. B. bought merchandise which he sold for $11275, and made thereby $2114: what was the cost price?

6. In 1870 the population of a country was 627413, and in 1880 it was 913279: what was the gain in 10 years?

7. The sum of two numbers is 9427, and the greater is 5825: what is the less number?

8. The minuend is 57, the subtrahend is 27: what is the remainder?

9. The minuend is 67, the remainder is 20: what is the subtrahend?

10. The subtrahend is 12, the remainder is 18: what is the minuend?

11. A gentleman gave to his son $3862, and to his daughter $5324: how much more did he give to his daughter than to his son?

12. In an orchard there are 425 apple-trees, and 297 plum-trees: how many more apple-trees are there than plum-trees?

13. A man travelled 14637 miles during one year and 9843 miles the next year: how much farther did he travel the one year than the other?

14. A merchant had 25694 pounds of pork, and sold 19832 pounds of it: how many pounds remained unsold?

15. A speculator bought a quantity of cotton for $294682, and sold it for $516390: how much did he gain?

16. A man owning 45761 acres of land, sold 23927 acres of it: how many acres had he remaining?

17. A merchant having 98072 barrels of flour, sold 49267 of them: how many had he remaining?

18. A certain town had 24967 inhabitants, which was 5084 more than it had the preceding year: how many had it the preceding year?

19. A merchant sold a quantity of goods for $38967, which was $873 more than they cost him: how much did they cost him?

20. A man having 21695 feet of umber, sold 7962 feet of it: how many feet had he remaining?

21. If I borrow of my neighbour $9673, and pay him $999 of it: how much remains unpaid?

22. A man has a farm of 400 acres; part is woodland, and part is cultivated; the former part is 125 acres: how much is the latter?

23. The distance from the earth to the sun is about 95000000 miles; the distance to the moon is about 240000: how much farther is it to the sun than to the moon?

24. If I bought a ship for $42650, and sold it for $49000: what did I gain?

25. A gentleman gave $12462 for a house and some land; the house alone was worth $9375: what was the value of the land?

26. A lumberman having 650000 feet of boards, sold 162372 feet of them: how many feet then remained?

27. The battle of Inkermann was fought in the year 1854; the Peninsular War was begun 46 years before this: in what year did the latter war begin?

28. A man having $100000, gave away $365: how much had he left?

29. A merchant owns property to the amount of $45563, and owes $21209: how much is he really worth?

30. If two candidates for office received in the aggregate 73462 votes, and the successful one had 45309 votes; how many did the other have?

31. One province contains 55405 square miles, and another 50914 square miles: how many more square miles does the one contain than the other?

32. Mount Sorata, in South America, is 24812 feet high, and 21241 feet higher than Mount Snowdon, in Wales: how high is Mount Snowdon?

33. Bonaparte was declared emperor in the year 1804, when he was 35 years of age: in what year was he born?

34. Sir Isaac Newton was born in the year 1642, and died in 1727: how old was he when he died?

35. Glass windows began to be used in private houses in England in the year 1180, which was 516 years after the discovery of glass by a monk named Benall: in what year was the discovery of glass made?

36. Gunpowder was invented by Swartz in the year 1330: how long was this before the invention of printing, which was in 1440?

37. Find a number to which, if thirty-four thousand three hundred and twenty-six be added, the result will be three million nineteen thousand and five?

38. Cotopaxi, the highest volcano in the world, is 18875 feet high: how much higher is it than Mount Etna, in Sicily, which is 10950 feet high?

39. Sorata, the highest land in America, is 24812 feet high: how many feet higher is it than Mount Ararat, which is 17000 feet high?

40. The highest land in North America is Mount St. Elias, which is 15000 feet high, and this mountain is 8712 feet higher than Mount Washington in New Hampshire: how high is Mount Washington?

41. St. Peter's Church at Rome, which is 450 feet high, is 157 feet higher than St. James' Cathedral, Toronto: what is the height of the Cathedral?

42. A man willed to his sons $17496, which was $3829 more than he willed to his daughters: how much did he will to his daughters?

43. A man owning 25721 acres of land, sold 19395 acres of it: how many acres had he remaining?

44. A merchant having 78004 barrels of flour, sold 49726 of them: how many barrels had he remaining?

45. John and James played marbles, the former having 49 and the latter 73: how many will each have when James has won 17 marbles from John?

MULTIPLICATION.

69. The numbers that were added together in the examples in Addition were nearly always different. We now come to a short method of adding together numbers that are the same.

Ex. 3+3+3+3=12.

In this example we have the number 3 taken 4 times, giving 12 as the sum; but instead of finding this sum by the usual process of addition, we obtain the same by saying 4 times 3 are 12.

Again, 5+5+5+5+5+5=30, which result is the same as saying 5 taken 6 times gives 30, or 6 times 5 are 30.

70. This produces a Table, which may be obtained by ordinary Addition, for

1	+	1	=	2	or	twice	1	are	2
2	+	2	=	4		"	2	"	4
3	+	3	=	6		"	3	"	6
4	+	4	=	8		"	4	"	8
5	+	5	=	10		"	5	"	10
6	+	6	=	12		"	6	"	12
7	+	7	=	14		"	7	"	14
8	+	8	=	16		"	8	"	16
9	+	9	=	18		"	9	"	18
10	+	10	=	20		"	10	"	20
11	+	11	=	22		"	11	"	22
12	+	12	=	24		"	12	"	24

Again,

1	+	1	+	1	=	3	or three times	1	are	3
2	+	2	+	2	=	6	" "	2	"	6

In the same way **four** times 6 will be found to be 24; **five** times 7 will be 35, etc.

71. These results will now be placed in the form of a Table, called the Multiplication Table, which must be accurately memorized by the pupil.

MULTIPLICATION TABLE.

Once			Twice			3 times			4 times		
1	is	1	1	are	2	1	are	3	1	are	4
2	"	2	2	"	4	2	"	6	2	"	8
3	"	3	3	"	6	3	"	9	3	"	12
4	"	4	4	"	8	4	"	12	4	"	16
5	"	5	5	"	10	5	"	15	5	"	20
6	"	6	6	"	12	6	"	18	6	"	24
7	"	7	7	"	14	7	"	21	7	"	28
8	"	8	8	"	16	8	"	24	8	"	32
9	"	9	9	"	18	9	"	27	9	"	36
10	"	10	10	"	20	10	"	30	10	"	40
11	"	11	11	"	22	11	"	33	11	"	44
12	"	12	12	"	24	12	"	36	12	"	48

5 times			6 times			7 times			8 times		
1	are	5	1	are	6	1	are	7	1	are	8
2	"	10	2	"	12	2	"	14	2	"	16
3	"	15	3	"	18	3	"	21	3	"	24
4	"	20	4	"	24	4	"	28	4	"	32
5	"	25	5	"	30	5	"	35	5	"	40
6	"	30	6	"	36	6	"	42	6	"	48
7	"	35	7	"	42	7	"	49	7	"	56
8	"	40	8	"	48	8	"	56	8	"	64
9	"	45	9	"	54	9	"	63	9	"	72
10	"	50	10	"	60	10	"	70	10	"	80
11	"	55	11	"	66	11	"	77	11	"	88
12	"	60	12	"	72	12	"	84	12	"	96

9 times			10 times			11 times			12 times		
1	are	9	1	are	10	1	are	11	1	are	12
2	"	18	2	"	20	2	"	22	2	"	24
3	"	27	3	"	30	3	"	33	3	"	36
4	"	36	4	"	40	4	"	44	4	"	48
5	"	45	5	"	50	5	"	55	5	"	60
6	"	54	6	"	60	6	"	66	6	"	72
7	"	63	7	"	70	7	"	77	7	"	84
8	"	72	8	"	80	8	"	88	8	"	96
9	"	81	9	"	90	9	"	99	9	"	108
10	"	90	10	"	100	10	"	110	10	"	120
11	"	99	11	"	110	11	"	121	11	"	132
12	"	108	12	"	120	12	"	132	12	"	144

72. **Multiplication** is, then, a short method of Addition, or the art of repeating one number as many times as there are units in another.

The number which is to be added or repeated is called the Multiplicand.

The number which shows the number of times the Multiplicand is to be repeated is called the Multiplier.

The final result is called the Product.

73. The sign of this operation is $\times$, and is read **multiplied by.** Thus, $11 \times 6 = 66$, would be read 11 **multiplied** by 6 equals 66. Here 11 is the **multiplicand**, for it is the number which is to be repeated **six times.** 6 is the **multiplier**, for it shews the number of times the **multiplicand** 11 is to be repeated. 66 is the product, for it shows the result of repeating 11 six times.

The pupil must clearly understand that while the result, 66, is supposed to be remembered from the Table, it may be obtained by adding, in the same way, six 11's together, that is,

11
11
11
11
11
11
—
66 *Ans.*

74. The following **Mental Questions** will test the pupil's knowledge of the **Multiplication Table:**

Ex.—What will 8 peaches cost at 6 cents apiece?
Ans. 48 cents.

Since 1 peach costs 6 cents, 8 peaches must cost 6 cents repeated 8 times, or 8 times 6 cents, which will be 48 cents.

Ex. 2.—If a barrel of flour costs 7 dollars, what will 9 barrels cost? *Ans.* 63 dollars.

Since 1 barrel costs 7 dollars, 9 barrels must cost 9 times 7 dollars, that is, 63 dollars.

☞ The product must always be of the same kind as the multiplicand, for it is only the multiplicand repeated a certain number of times.

The best practice is to take the different products in the Multiplication Table, and give the numbers that are multiplied together to produce these products.

Ex.—What numbers give 42? *Ans.* 6 and 7.
What numbers give 108? *Ans.* 12 and 9.

EXERCISE 18.

(*a*) 1. Copy on your slates and fill out the following:

7 × 5 =	3 × 2 =	9 × 8	9 × 4 =
6 × 3 =	2 × 1 =	9 × 5	9 × 0 =
5 × 3 =	4 × 1 =	9 × 3	10 × 5 =
8 × 6 =	5 × 3 =	7 × 2	12 × 1 =
8 × 4 =	6 × 1 =	8 × 2	10 × 10 =
9 × 6 =	9 × 7 =	6 × 2	11 × 9 =
12 × 8 =	11 × 5 =	12 × 10	8 × 5 =
12 × 11 =	10 × 6 =	12 × 12	11 × 3 =
10 × 8 =	12 × 9 =	12 × 6	12 × 2 =
11 × 7 =	12 × 7 =	12 × 4	12 × 3 =
3 × 1 =	4 × 3 =	4 × 2	5 × 4 =
5 × 1 =	5 × 2 =	6 × 4	7 × 4 =
6 × 5 =	7 × 1 =	7 × 3	8 × 5 =
7 × 6 =	8 × 1 =	8 × 3	10 × 1 =
8 × 7 =	9 × 2 =	10 × 2	10 × 7 =
10 × 4 =	10 × 3 =	10 × 5	11 × 4 =
10 × 9 =	11 × 1 =	11 × 2	12 × 5 =
11 × 5 =	12 × 8 =	12 × 6	12 × 10 =

2. Repeat all the numbers of times 5, from once 5 to 10 times 5. Thus, once 5 is 5, 2 times 5 are 10, 3 times 5 are 15, etc.

3. Repeat from once 6 to 10 times 6, and back from 10 times 6 to once 6.

4. Repeat from once 7 to 10 times 7, and back.

5. Repeat from once 8 to 10 times 8, and back.

6. Repeat from once 9 to 10 times 9, and back.

7. Repeat from once 10 to 10 times 10, and back.

8. What two numbers produce by multiplication the following numbers: 2, 8, 12, 40, 60, 72, 12, 11, 22, 36, 24, 44, 54, 77, 81, 96, 55, 84, 108, 88, 132, 120, 64, 49, 56, 63, etc.

(*b*) 1. A quart of berries makes 2 pints: how many pints in 8 quarts?

2. There are 12 inches in one foot length of rope: how many inches in 9 such lengths?

3. If 1 lemon costs 7 cents, how many cents must you pay for 4 lemons?

4. One gallon contains 4 quarts: how many quarts are there in 11 gallons?

5. One peck contains 8 quarts: how many quarts are there in 7 pecks?

6. What would 12 pears cost at 7 cents apiece?

7. How much would 5 bags of meal cost at 9 dollars a bag?

8. At 10 cents a pint, how much would 8 pints of cherries cost?

9. At 11 cents a pound, how much would 7 pounds of sugar cost?

10. In one week there are 7 days: how many days are there in 11 weeks?

11. At 8 cents a quart, what would be the cost of 7 quarts of berries?

12. In 1 florin there are 45 cents: how many cents would there be in 7 florins?

13. At 3 dollars a bushel, what would 12 bushels of grapes cost?

14. There are 4 gills in one pint: how many gills would there be in 12 pints?

15. If a horse runs 12 miles an hour, how far would he run in 5 hours?

16. In one yard there are 3 feet: how many feet in 8 yards?
17. At 6 cents a pound, how much must you pay for 9 pounds of oatmeal?
18. In one foot there are 12 inches: how many inches in 11 feet?
19. How much would 8 pounds of metal cost at 6 cents a pound?
20. At 8 cents apiece, how much would you pay for 8 pencils?
21. If one box holds 3 bushels of fruit, how many bushels would 10 boxes hold?
22. If a ton of coal cost 9 dollars, what would 10 tons cost?
23. At 11 cents an ounce, what would 9 ounces of blue be worth?
24. Six feet make a fathom: how many feet would there be in 9 fathoms?
25. If a bunch of grapes costs 7 cents, what would 9 bunches cost?
26. In one mile there are 8 furlongs: how many furlongs would there be in 12 miles?
27. At 10 cents a gill, how much would 11 gills of wine cost?
28. In one bushel there are 4 pecks: how many pecks are there in 8 bushels?
29. At 3 cents a box, how many cents must you pay for 7 boxes of matches?
30. There is a garden of six rows of tulips, with six tulips in a row: how many tulips are there in the garden?
31. Each step in a flight of stairs is eleven inches high: how many inches will you ascend in eleven steps?
32. If you work eleven examples each day, how many will you work in six days?
33. How many are seven times seven pounds?
34. There are seven days in one week: how many days are there in six weeks?

35. What will be the cost of seven pounds of raisins at nine cents a pound?

36. A sheet of paper can be folded so as to make four leaves: how many leaves will eleven such sheets make?

37. What is the cost of eight pounds of soap at seven cents a pound?

38. There are twelve inches in a foot: how many inches are there in twelve feet?

39. How many inches in eight feet?

40. If one barrel of flour costs nine dollars, how much will eight barrels cost?

41. If one shot weighs six pounds, what is the weight of seven such shot?

42. What is the weight of eight packages of coffee, if each weighs five pounds?

43. If one pound of rice costs twelve cents, what is the cost of seven packages of a pound each?

44. What is the cost of twelve pair of boots at six dollars a pair?

45. I bought eleven pounds of glue at eight cents a pound what was the cost?

46. William had six cents; his sister gave him three more, and his mother gave him seven times as many as he then had: how many did his mother give him?

---

75. The pupil is now supposed to be quite familiar and ready with the Multiplication Table, and we will therefore go on with the different cases that occur in multiplication.

First we will multiply any number by a number of one figure, or by any number from 1 to 12.

Ex.—Multiply 3781 by 7.

Multiplicand.	3781
Multiplier.....	7
Product.....	26467

Place the multiplier 7 under the multiplicand 3781, and begin to multiply from the right, or units' place. 7 times 1 unit are 7 units, which we place in the usual place. 7 times 8 tens are 56 tens, that is 5 hundreds and 6 tens; place the 6 tens in its proper place, and carry the 5 hundreds. 7 times 7 hundreds are 49 hundreds, which, with the 5 hundreds we carried, make 54 hundred, or 5 thousands and 4 hundreds; place the 4 hundreds in its own place, and carry the 5 thousands. 7 times 3 thousands are 21 thousands, and the 5 thousands we carried make 26 thousands, which is placed on the left.

76. This may be done more easily by neglecting for the time the words **units, tens,** etc.

Thus: 7 times 1 are 7, put down the 7; 7 times 8 are 56, put down the 6 and carry 5; 7 times 7 are 49, and 5 (carried) make 54, put down the 4 and carry the 5; 7 times 3 are 21, and the 5 (carried) make 26.

The pupil should be satisfied that the same result could have been obtained by Addition, thus:

$$\begin{array}{r} 3781 \\ 3781 \\ 3781 \\ 3781 \\ 3781 \\ 3781 \\ 3781 \\ \hline 26467 \end{array}$$

77. Thus, when the multiplier does not exceed 12, we have the following

RULE FOR MULTIPLICATION.

Place the Multiplier under the Multiplicand; begin at the units' place, and multiply each figure in the Multiplicand by the Multiplier. Set down the right-hand figure of the product, and carry the left-hand figure or figures, if any, to the next product—just as in Addition.

EXERCISE 14.

(a) 1. Multiply 123 by 2.
2. Multiply 134 by 2.
3. Multiply 223 by 2.
4. Multiply 246 by 2.
5. Multiply 278 by 2.
6. Multiply 495 by 2.
7. Multiply 1312 by 2,
8. Multiply 2172 by 2.
9. Multiply 3629 by 2.
10. Multiply 3785 by 2.
11. Multiply 4006 by 2.
12. Multiply 4308 by 2.
13. Multiply 142034 by 2.
14. Multiply 1706324 by 2.
15. Multiply 3614503 by 2.
16. Multiply 462178 by 2.
17. Multiply 1203062 by 3.
18. Multiply 21607835 by 3.
19. Multiply 93420 by 3.
20. Multiply 705086 by 3.
21. Multiply 1039246 by 4.
22. Multiply 217906 by 4.
23. Multiply 509367 by 4.
24. Multiply 567239 by 5.
25. Multiply 6146802 by 6.
26. Multiply 4601792 by 5.
27. Multiply 962078 by 6.
28. Multiply 729360 by 7.
29. Multiply 4286072 by 7.
30. Multiply 237000 by 7.
31. Multiply 23416 by 2.
32. Multiply 45613 by 4.
33. Multiply 34520 by 3.
34. Multiply 56042 by 5.
35. Multiply 21264 by 2.
36. Multiply 60153 by 6.
37. Multiply 32305 by 5.
38. Multiply 57802 by 2.
39. Multiply 80135 by 5.
40. Multiply 91246 by 6.
41. Multiply 12357 by 7.
42. Multiply 23460 by 6.
43. Multiply 68913 by 3.
44. Multiply 57802 by 2.
45. Multiply 62819 by 5.
46. Multiply 93856 by 6.
47. Multiply 28475 by 4.
48. Multiply 39586 by 5.
49. Multiply 40697 by 6.
50. Multiply 17364 by 3.
51. Multiply 51708 by 4.
52. Multiply 5876 by 4.
53. Multiply 8546 by 7.
54. Multiply 502 by 9.
55. Multiply 246025 by 5.
56. Multiply 512604 by 8.
57. Multiply 648 by 7.
58. Multiply 1082 by 9.
59. Multiply 5050 by 4.
60. Multiply 73046 by 3.
61. Multiply 10708 by 2.
62. Multiply 980789 by 8.

(63)	(64)	(65)	(66)	(67)
2413	3142	3546	5354	81897
11	12	11	12	11

(68)	(69)	(70)	(71)	(72)
900867	8602968	716914	765439	8419829
11	12	12	11	11

(*a*)

(73)	(74)	(75)	(76)	(77)
8270802	3443	1345	77777	888883
12	11	12	11	12

(78)	(79)	(80)	(81)	(82)
666666	9999999	4040404	575757	484848
10	12	11	12	11

83. 82386 × 7.	96. 18889 × 12.	109. 960281 × 11.
84. 357 × 5.	97. 18476 × 11.	110. 593514 × 8.
85. 8645 × 8.	98. 437958 × 7.	111. 926847 × 12.
86. 2079 × 9.	99. 760281 × 11.	112. 760281 × 9.
87. 8842 × 4.	100. 194514 × 8.	113. 104748 × 7.
88. 3749 × 7.	101. 426847 × 12.	114. 327071 × 11.
89. 13146 × 9.	102. 859170 × 9.	115. 650304 × 8.
90. 876 × 10.	103. 482403 × 7.	116. 382637 × 12.
91. 2345 × 12.	104. 715736 × 11.	117. 616960 × 9.
92. 998 × 11.	105. 548069 × 8.	118. 438082 × 7.
93. 8134 × 12.	106. 871392 × 12.	119. 871406 × 11.
94. 7312 × 11.	107. 204625 × 9.	120. 763867 × 12.
95. 8183 × 12.	108. 537958 × 7.	

(*b*) 1. A man sold 195 lambs at $3 apiece: how much did he receive for them?

2. What is the cost of 184 barrels of meal, at $6 a barrel?

3. What is the cost of 1987 acres of ground, at $9 an acre?

4. What is the cost of 4786 barrels of flour, at $9 a barrel?

5. In 1 mile there are 5280 feet: how many feet are there in 5 miles?

6. In 1 mile there are 1760 yards: how many yards are there in 5 miles?

7. If 9 men can sow a farm in 18 days, in how many days can one man do the same?

8. If 6 men can build a wall in 149 days, in how many days can one man build the same wall?

9. If 460 bushels of grain will feed one horse 18 months: how many bushels will be necessary to feed 8 horses for the same time?

10. I bought 245 cords of maple, at $7 a cord: how much did the whole cost me?

11. A dealer sold 8 animals, at $253 apiece: how many dollars did he receive for them?

12. A girl bought 189 yards of ribbon at 6 cents per yard: how much did it cost her?

13. What is the cost of 2988 boxes of figs, at $3 a box?

14. If a steamer can go 395 miles in one day, how far can she go in 9 days at the same rate?

15. There are 10 companies in the Queen's Own Rifles, Toronto, each having 42 men: how many men are there in the regiment?

16. In one mile there are 5280 feet: how many feet are there in 4 miles?

17. If a mill turns out 9757 yards of carpet in one week, how many yards could it produce in 5 weeks?

18. If a ship can carry 7856 barrels of ore, how many barrels could be carried in 6 ships?

19. If a waggon can carry 5837 shingles, how many shingles could be carried in 7 waggons?

20. What would 8 miles of pavement cost, at $3489 per mile?

21. Nine men built a vessel, each one putting in $8457: what was the cost of the vessel?

22. If a barge can carry 19857 pounds, how many pounds could 4 such vessels carry?

23. There are 63360 inches in a mile: how many inches are there in 5 miles?

24. How many miles would a yacht sail in going around the earth 6 times, the earth being 24856 miles in circumference?

25. What would be the cost of constructing 7 miles of embankment, at $35248 per mile?

26. If 11 clergymen are paid $2212 dollars apiece in Toronto, what do they receive in all?

27. What is paid to 12 teachers in Hamilton, at the rate of $862 each?

28. In 1 mile there are 5280 feet: how many feet are there in 7 miles?

29. In 1 mile there are 1760 yards: how many yards are there in 8 miles?

30. If in one barrel there are 196 pounds of meal, how many pounds are there in 5 barrels?

31. A. sold 4 horses, at $175 apiece: how many dollars did he receive for them?

32. A lady bought 89 yards of silk twist, at 5 cents a yard: how much did the twist cost her?

33. What is the cost of 1786 boxes of grapes, at $2 a box?

34. If a vessel can go 387 miles in 1 day, how far can she go in 9 days at the same rate?

35. A merchant bought 1789 coats, at $4 apiece: how much did he pay for them all?

36. A farmer sold 7 fat cattle for $97 apiece: how much did he receive for them all?

37. In 1 gallon there are 4 quarts: how many quarts will there be in 8451 gallons?

38. There are 160 square rods in 1 acre of land: how many square rods are there in 11 acres?

39. How many pence are there in 32916 shillings, at 12 pence to the shilling?

40. In 1 year there are 365 days: how many days are there in 11 years?

☞ In the question, "Bought 149 square yards of land at $7 a yard," although we have to find what $7 become when repeated 149 times, yet we may, for the sake of convenience, look upon 7 as the multiplier, the product being dollars.

---

78 The next case of multiplication is that in which the multiplier is greater than 12.

Ex. 1.—Multiply 1396 by 364, that is, find what 1396 becomes when repeated 364 times.

			1396
			364
1396 × 4 =	5584		5584
1396 × 60 =	83760	Or,	8376
1396 × 300 =	418800		4188
	508144		508144

The number 364 = 3 hundreds + 6 tens + 4 units, hence the multiplicand is to be repeated 300 times, and 60 times, and 4 times. If we then take 4 times the multiplicand, and 60 times the multiplicand, and 300 times the multiplicand, these results added together must give 364 times the multiplicand.

By the previous case, 4 times 1396 gives 5584. This we put down as usual.

Again, 1396 multiplied by 6 tens is the same as 6 tens by 1396 (Art. 77), and this we find to be 8376 tens, which result in the addition must (since it is tens) be put one place to the left of the last result, 5584.

Again, 1396 multiplied by 3 hundreds is the same as 3 hundreds by 1396, and this is 4188 hundreds, which must therefore in the addition be put one place to the left of the tens' result, 8376.

Having placed these three results ready for addition, nothing remains but to add them together in the usual way.

We thus find that $1396 \times 364 = 508144$.

In the same way we proceed with any number of figures in the multiplier.

Ex. 2.—Multiply 872 by 307.

			872
			307
872 × 7 =	6104		
872 × 0 =	00	Or,	6104
872 × 300 =	261600		26160
	267704		267704

Here 7 times 872 gives 6104. There are no tens in the multiplier, hence we might have filled the usual line with noughts, but one nought is enough to keep the next result in its proper place. 3 times 872 gives 2616, and this being hundreds, it must be put one place to the left of the nought, which makes the tens' place.

Add as before, and we find $872 \times 307 = 267704$.

Ex. 3.—Multiply 371 by 2100.

$$\begin{array}{r} 371 \\ 2100 \\ \hline 37100 \\ 742 \\ \hline 779100 \end{array}$$

In this example there are no units or tens in the multiplier, therefore the first result, 371, must be placed to represent hundreds, that is, three places to the left of the units' place. The next result, being thousands, viz., 742 thousands, it will be put one place to the left of the last result. Add together as usual.

79. We see from this that any number may be multiplied by 10, 100, 1000, etc., by adding 1, 2, 3, etc., noughts to the right of the number to be multiplied.

Thus, $389 \times 100 = 38900$.
$40 \times 1000 = 40000$.

80. As there are 100 cents in a dollar, this principle is very useful in expressing any number of dollars as cents.

Ex. 1.—How many cents are there in \$84?

There will be 84 times as many cents in 84 dollars as there are cents in 1 dollar, that is, $84 \times 100 = 8400$ cents.

Ex. 2.—How many cents are there in \$64.52; that is, 64 dollars and 52 cents?

\$64 = 6400 cents.
6400 cents + 52 cents = 6452 cents.

81. Hence, to express any number of dollars and cents as cents, we have only to remove the point which separates the dollars from the cents, and the result will be the required number of cents.

Ex. 3.—\$106.97 = 10697 cents.

82. Suppose we have to multiply any number by 16. We know that $8 \times 2 = 16$, and therefore to repeat the number 16 times would amount to the same as repeating it 8 times and then repeating this result 2 times; or, since $2 \times 8 = 16$, it would be the same to repeat the number 2 times and then repeat that result 8 times.

Again, $4 \times 4 = 16$. We may therefore repeat the number 4 times, and this result again 4 times; each of these methods would give the same product.

Ex.—Multiply 362 by 44.

$$\begin{array}{r} 362 \\ 11 \\ \hline 3982 \\ 4 \\ \hline 15928 \end{array}$$

Since $44 = 11 \times 4$, we multiply 362 by 11, which gives 3982, and then multiply 3982 by 4, giving 15928 as the final product.

83. The numbers 8 and 2, or 4 and 4, are called the factors of 16. (See Art. 74.)

The factors of		
12	are	2, 6.
48	are	12, 4.
120	are	12, 10.

84. A result in Multiplication may be proved to be correct by using the multiplicand as the multiplier, which should give the same product, if worked correctly.

EXERCISE 15.

☞ As many as possible of the following questions should be worked by factors as well as by the ordinary method.

	(1)	(2)	(3)	(4)
(*a*)	4624	3846	8462	7846
	35	39	47	147

(a)

(5)	(6)	(7)	(8)
3976	2243	763521	1283
183	144	438	144

(9)	(10)	(11)	(12)
2526	52365	3678543	767853 16
136	543	4567	7615

(13)	(14)	(15)	(16)
67854	6503456	98610275	568
10234	234	35789	287

(17)	(18)	(19)	(20)
3985	987	7415	8097
733	891	387	869

(21)	(22)	(23)	(24)
57423	194	3678543	437
159	57	4567	356

(25)	(26)	(27)	(28)
274	43326	999	841
167	96	999	841

(29)	(30)	(31)	(32)
3759	8643	3976	907
3757	923	948	740

(33)	(34)	(35)	(36)
657	6258	5679	7856
408	346	507	658

(37)	(38)	(39)	(40)
9008	3207	6579	8579
784	2345	3506	4078

(41)	(42)	(43)	(44)
7058	35768	726	4628
6007	3456	27	554

(a)

(45)	(46)	(47)	(48)
3648	4275	8463	53642
36	54	3759	63

(49)	(50)	(51)	(52)
4620	8726	7692	2146
524	463	356	179

(53)	(54)	(55)	(56)
37642	37942	27403	81650
57	386	584	789

(b) **Multiply**

1.	74	by 10.	29.	2572	by 94.
2.	10000	by 869.	30.	40306	by 127.
3.	4698	by 1000.	31.	86072	by 208.
4.	100000	by 76984.	32.	48746	by 316.
5.	6307918	by 20790.	33.	30975	by 507.
6.	44670	by 145.	34.	6408	by 325.
7.	367950	by 756.	35.	703475	by 386.
8.	78609	by 406.	36.	370607	by 4071.
9.	887002	by 903.	37.	600326	by 2645.
10.	684207	by 7006.	38.	730096	by 5006.
11.	7532100	by 4861.	39.	2407068	by 3406.
12.	17565	by 1800.	40.	408091	by 2407.
13.	43450	by 1700.	41.	73069	by 46035.
14.	685900	by 190.	42.	4372	by 128.
15.	76980	by 16000.	43.	3065	by 84.
16.	78600	by 1400.	44.	36204	by 414.
17.	708060	by 490.	45.	4008	by 3724.
18.	43800	by 38506.	46.	47672	by 234.
19.	70800	by 69870.	47.	302076	by 603.
20.	1011	by 754.	48.	73008	by 2036.
21.	10009	by 869.	49.	430605	by 4005.
22.	386	by 99.	50.	290361	by 30406.
23.	7815	by 98.	51.	2784	by 216.
24.	6188	by 97.	52.	68470	by 435.
25.	7289	by 999.	53.	3060724	by 2406.
26.	38751	by 998.	54.	130065	by 8042.
27.	67583	by 996.	55.	98070	by 12094.
28.	74189	by 995.	56.	6789	by 2345

(c) 1. Multiply seven thousand six hundred and one by seven.

2. Multiply thirteen hundred and eighty-four by eleven and twelve in succession.

3. Multiply together two, three, four, five, six, seven, eight, nine, and ten.

4. Find the product of three hundred and forty-seven and five hundred and eighty three.

5. Find the product of twelve thousand and three and three thousand and twelve.

6. How much is twelve times four times three thousand four hundred and seven?

7. Multiply together three thousand three hundred, three thousand and thirty, and three thousand and three.

8. Find the **square** of six hundred and seventy-nine.

☞ (The **square** of a number is that number multiplied by itself.)

9. Find the square of two thousand seven hundred and forty-seven.

10. Find the square of seventeen hundred.

11. Multiply three millions seventeen thousand and ninety by four thousand and eighty-four.

12. Multiply the square of two hundred and thirty-nine by eleven.

13. Find the product of one thousand three hundred and fifty-six, five hundred and seventy-eight, and two hundred and fifty.

14. Multiply the square of seventeen by the square of nineteen.

15. Multiply six thousand and ninety-seven by nine hundred and eight.

16. Multiply fifty-four thousand and forty-nine by six thousand and seventy-five.

17. The two factors of a certain number are 656 and 907: what is the number?

18. Multiply thirty-seven thousand and twenty-eight by 508.

19. The multiplier being 987, the multiplicand six thousand four hundred and sixteen: required the product.
20. What is the product of 908060 multiplied by five thousand four hundred?
21. One factor is 718151, the other seven hundred: what is the product?
22. Multiply six hundred and seventy-four thousand two hundred by two thousand one hundred and four.
23. Multiply ninety-three thousand one hundred and eighty-six by four thousand four hundred and fifty-five.
24. How many cents are there in $5? in $60? in $18? in $47? in $22.05? in $872.06? in $540.10? in $80.80?
25. How many more cents are there in $20 than in $12.75?
26. A. earns $1.75 in a day, and B. earns $1.60: how many more cents does A. earn in 6 days than B.?
27. A man has $860.75, and lends another $851: how many cents has he left?
28. How many cents are there in 10 times $60?
29. How many more cents are there in 3 times $17 than in 4 times $3.25?
30. A owns $570.60 in stock, and buys 3 times as much more: how many cents will he then have invested in stock?

(*d*) 1. If an orchard containing 313 trees yields 15 bushels of apples to a tree, how many bushels does the whole orchard produce?
2. How many panes of glass are there in 18 windows, if each window contains 24 panes?
3. How many bushels of wheat will 160 acres produce, at the average rate of 45 bushels to the acre?
4. What is the cost of 2463 barrels of flour, at 19 dollars a barrel?
5. If a man can earn 83 dollars in one month, how many dollars can he earn in 18 months?
6. What will be the cost of an estate containing 684 acres, at 57 dollars per acre?
7. A garden has 625 hills of potatoes, and each hill will average 13 potatoes: how many potatoes at that rate will there be in the garden?

8. If 17 men do a piece of work in 91 days, how long will it take one man alone to do it?

9. What will be the cost of building a line of telegraph 274 miles long, at $967 a mile?

10. If 1049 pounds of tobacco can be raised from an acre of land, how many pounds will 386 acres produce?

11. If a mill turns out 628 yards of cloth in a day, how many yards can it make in 297 days?

12. What will be the cost of building 279 miles of railroad, at $27384 a mile?

13. A crop of cotton was put up in 340 bales, each bale containing 596 pounds: what was the weight of the entire crop?

14. What is the value of 108 buildings, at $1896 each?

15. What is the cost of 257 yoke of oxen, at $175 a yoke?

16. What is the cost of 428 lots, at $284 each?

17. In 1 ream of paper there are 480 sheets: how many sheets are there in 217 reams?

18. How much will it cost to build a line of telegraph from Toronto to Lake Nipissing, the distance being 236 miles, at $1270 a mile?

19. If a cotton mill manufacture 658 yards of cloth in a day, how many yards can it make in 309 days?

20. How many yards of cloth are in 265 pieces, each piece containing 32 yards?

21. Sound travels 1142 feet in 1 second: how far will it travel in 60 seconds?

22. Light travels 192000 miles in 1 second: how far will it travel in 494 seconds?

23. A drover bought 685 oxen, at 10_{4}$ apiece what was the cost of all of them?

24. A merchant bought 25 pieces of broadcloth, each piece containing 48 yards, at $9 a yard: how much did he pay for the whole?

25. If the *Thunderer* can steam 18 miles in 1 hour, how far can she steam in 34 days of 24 hours each?

26. A man bought 8969 acres of land, at $196 an acre: how much did the whole cost him?

27. In 1 furlong there are 660 feet: how many feet are in 8 furlongs (1 mile)?

28. How many pounds of pork are there in 395 barrels, there being 200 pounds in each barrel?

29. What is the value of 346 shares of bank stock, at $125 a share?

30. How many pages are there in 5896 books, there being 394 pages in each book?

31. A speculator bought 302 cattle, and 293 times as many sheep: how many sheep did he buy?

32. If a body move at the rate of 378 miles a day, how far would it move in 365 days?

33. What is the cost of 1787 barrels of sugar, at 18 dollars a barrel?

34. What is the value of 1982 barrels of molasses, at 15 dollars a barrel?

35. What is the cost of 3784 pieces of broadcloth, at 143 dollars apiece?

36. What will I be charged for 21423 barrels of pork, at 23 dollars a barrel?

37. What must I pay for 47879 bushels of corn, at 65 cents a bushel?

38. How many dollars would purchase 3785 kegs of tobacco, at 34 dollars a keg?

39. At 19 dollars a firkin, what is the cost of 91072 firkins of butter?

40. If 7842 men build a fort in 137 days, how long would it take 1 man to build it?

41. What will be the value of 237 cows, at 23 dollars each?

42. What will be the cost of 397 loads of metal, at 37 dollars a load?

43. What is the cost of 2473 tons of wrought iron, at 297 dollars a ton?

44. If a regiment consists of 1128 men, how many men are there in an army of 203 regiments?

45. In a factory there are 873 yards of cloth made in 1 day: how many yards, at this rate, can be made in 313 days?

46. In one load a span of horses can draw 2997 pounds how many pounds would they draw in 327 loads?

47. There are 15 fields of plants; in each field there are 97 rows, and 256 plants in each row: how many plants are there in all?

48. How many letters are there in a book containing 672 pages, each page containing 43 lines, and each line 47 letters?

49. A freight train consists of 21 cars; each car contains 85 barrels, and each barrel weighs 196 pounds: how many pounds are in the entire cargo?

50. The distance from Toronto to Thornhill is 12 miles; each mile contains 1760 yards, and each yard 3 feet: how many feet are there from one place to the other?

51. In an orchard there are 14 rows of plum trees; each row contains 27 trees, and each tree bears 108 plums, how many plums are in the orchard?

52. It requires 1716 rails to fence one side of a square garden: how many rails will be required to fence 13 lots of the same size and shape?

53. An army lost in battle 315 killed and 417 wounded; the enemy lost altogether 13 times as many: how many soldiers were killed and wounded in the battle?

54. If two steamers should leave Collingwood at the same time, and should sail in the same direction, the first at the rate of 18 miles an hour, the second at the rate of 15 miles an hour, how far apart would they be in 36 hours?

55. An army consists of 6 divisions, each division of 4 battalions, and each battalion of 613 men: find the number of men in the army.

56. If a planing-mill run 4360 feet of boards a day how many will it run out in 106 days?

57. Supposing the earth to move around the sun at the rate of 68000 miles an hour, how far will it move in 365 days of 24 hours each?

58. Bought from Davison, Scott & Co. 427 cheeses, weighing, with the cases, 67 pounds each; each case weighs 5 pounds: find the cost of the cheese at 11 cents a pound.

59. On a holiday in theCity of Toronto, the Street Railway Company placed on the road 76 cars; the fare is 5 cents: supposing each to carry, on an average, 18 persons, and to make 10 trips, how much money was taken by the Company on that day?

---

85. The following mental exercises will be found useful in accustoming the pupils to rapid thought.

The questions should be read out slowly at first, but gradually faster, and each pupil should write upon a slate or paper on the desk in front the result obtained.

☞ As this is the most valuable work that can be done by the beginner, the teacher should add largely to the number of problems here set.

Ex. 1.—Add 15 to 4, subtract 9, add 11, subtract 5: what is the result?

$15+4=19$; $19-9=10$; $10+11=21$; $21-5=16$.

Ans. 16.

While the teacher dictates the example, "to 15 add 4, subtract 9," etc., the pupils think 19, 10, etc.

Ex. 2.—Take 19, subtract 9, multiply by 7, subtract 11, subtract 9.

The pupil would think: 19, 10, 70, 59, 50. *Ans.* 50.

EXERCISE 16.

1. To 12 add 7, subtract 5, add 4, add 8: result?
2. From 25 take 10, add 7, add 8, take 9: result?
3. To 17 add 18, subtract 7, add 9, subtract 5: result?

4. To 26 add 18, subtract 5, add 6, subtract 9, add 5, take 8 : result?
5. From 27 take 9, add 7, subtract 10, subtract 6, add 11 : result ?
6. Add 9 to 15, subtract 11, add 10, subtract 9, subtract 14 : result ?
7. Take 4 from 23, add 1, add 25, subtract 5, subtract 20, add 6: result ?
8. Add 7 to 8, subtract 3, add 8, add 12, subtract 7, subtract 5, add 6, add 4, subtract 10 : result ?
9. Take 7 from 15, add 6, take 5, add 10, take 3, add 4, add 6, take 7, add 8, take 9 : result ?
10. To 13 add 7, subtract 5, multiply by 2, subtract 15, subtract 10, multiply by 3, subtract 5 : result?
11. From 15 subtract 9, multiply by 3, subtract 8, add 5, multiply by 2, subtract 20, add 8, multiply by 2, subtract 9, add 9 : result?
12. Multiply 12 by 5, subtract 40, add 5, multiply by 2, subtract 25, add 5, multiply by 3, add 7 : result ?
13. Take 12 from 48, add 6, take 7, add 8, take 3, add 7, take 9, add 11, take 4, add 3, take 5, add 9, take 10, add 7, take 8, add 9, add 3, take 5 : result ?
14. From 16 subtract 9, multiply by 3, subtract 7, add 4, multiply by 6, subtract 7, add 9, subtract 8 : result ?
15. Add 6 to 18, subtract 9, multiply by 4, subtract 25, multiply by 2, subtract 40, multiply by 7 : result ?
16. From 19 subtract 8, multiply by 6, subtract 11, add 7, subtract 20, add 8, multiply by 3, add 9: result ?
17. Multiply 7 by 6, subtract 12, add 4, subtract 14, multiply by 6, subtract 20, multiply by 3, subtract 12, add 12 : result?
18. Add 11 to 29, multiply by 2, subtract 16, add 6, multiply by 10 : result?
19. Take 19 from 39, multiply by 5, subtract 50, add 10, multiply by 3, subtract 100 : result ?
20. To 23 add 7, multiply by 3, add 10, subtract 50, multiply by 2, subtract 100, multiply by 6 : result?

21. Add 7 to 9, subtract 6, multiply by 4, subtract 20, add 7, subtract 5, multiply by 2, subtract 8, add 5 : result ?

22. Subtract 8 from 17, multiply by 5, subtract 15, multiply by 20, subtract 30, add 9, subtract 9 : result ?

23. To the product of 8 and 8 add 6, subtract 30, add 2, subtract 12, multiply by 3, subtract 4, add 6 : result ?

24. To 19 add 11, subtract 15, multiply by 4, subtract 12, multiply by 2, add 9, subtract 5, multiply by 11 : result ?

25. Subtract 9 from 21, add 8, subtract 6, add 11, multiply by 4, subtract 7, add 9, subtract 8 : result ?

26. To the product of 9 and 6 add 6, subtract 12, subtract 18, multiply by 2, subtract 20, add 5, multiply by 2, add 10, multiply by 3, add 15, subtract 7, add 6 : result ?

27. From 23 subtract 8, multiply by 2, multiply by 4, subtract 20, subtract 21, add 6, multiply by 2, subtract 20, multiply by 3, add 8, subtract 7, add 9 : result ?

28. To 31 add 12, subtract 10, add 6, subtract 7, subtract 8, subtract 4, add 9, subtract 3, add 6, add 8, add 10, subtract 5, add 8, subtract 2, add 6, subtract 7, add 4, subtract 6 : result ?

29. From 63 subtract 7, add 3, add 6, add 12, subtract 4, add 10, subtract 5, add 6, subtract 7, add 3, subtract 6, add 9, subtract 8, add 6, subtract 4, add 3, subtract 2, add 7 : result ?

30. Add 7 to 9, subtract 8, add 20, add 14, add 30, subtract 4, add 5, subtract 6, add 7, subtract 8, add 9, subtract 10, add 4, add 5, add 8, subtract 7, add 4, subtract 5 : result ?

DIVISION.

86. We have seen that 3 dollars repeated 5 times are 15 dollars. Now let us see how often we can take 3 dollars from 15 dollars.

15 dollars
3
—
12 dollars = 1st remainder.
3
—
9 " = 2nd "
3
—
6 " = 3rd "
3
—
3 " = 4th "
3
—
0 " = 5th "

Thus, 3 dollars may be taken 5 times away from 15 dollars, that is, just as many times as it was before repeated in order to produce 15 dollars.

This fact is expressed by saying that 3 is contained in 15, 5 times. In the same manner it may be shewn that 4 is contained in 12, 3 times; and 8 contained in 56, 7 times.

87. Again, since 3 dollars can be taken 5 times from 15 dollars, this is but another way of saying that $15 can be divided into 5 parts, each part being 3 dollars. In the same way, if 20 units be divided into 4 parts of the same size, each will be 5 units, and since, in repeating the parts in the multiplication, they were, of necessity, the same size, so in this process we will always suppose the parts to be the same size, or of the same value.

When we wish to divide 32 into 4 parts, it is understood that they shall be of the same size, viz: 8 units in this case.

88. This operation, then, is called **Division**, which is therefore the method of finding the number of times one number is contained in another.

The number which contains, or is divided by the other is known as the **Dividend.**

The number which is to be divided into the **Dividend** is called the **Divisor.**

The number which shows now often the **Divisor** is contained in the **Dividend** is termed the **Quotient.**

89. The sign for this operation is ÷, placed between the two numbers, and shews that the number coming **before it**, viz., the **Dividend**, is to be divided by the one coming after it, viz.: the **Divisor**, thus:

$$45 \div 9 = 5,$$

Reads, 45 divided by 9 equals 5, and means that 9 may be taken from 45, 5 times; or, that 9 is contained in 45, 5 times; or, that if 45 be divided into 9 **equal** parts, each part is 5.

☞ The pupil should remember that it is the **Divisor** which always follows the sign of **Division.**

90. Division will easily be seen to be the converse of Multiplication, for from 7 and 4 we obtained 28 by the latter process, while from 7 and 28 we obtain 4 by the former process.

Every result, then, in the Multiplication Table (Art. 71) will also furnish us with a corresponding result in the Division Table. This is, in fact, the very work done by the pupil in Ex. 13 (*a*).

91. The following Table can be seen at once to agree with the Table in Art. 71.

DIVISION TABLE.

1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9
5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
9 ÷ 9 = 1	10 ÷ 10 = 1	11 ÷ 11 = 1	12 ÷ 12 = 1
18 ÷ 9 = 2	20 ÷ 10 = 2	22 ÷ 11 = 2	24 ÷ 12 = 2
27 ÷ 9 = 3	30 ÷ 10 = 3	33 ÷ 11 = 3	36 ÷ 12 = 3
36 ÷ 9 = 4	40 ÷ 10 = 4	44 ÷ 11 = 4	48 ÷ 12 = 4
45 ÷ 9 = 5	50 ÷ 10 = 5	55 ÷ 11 = 5	60 ÷ 12 = 5
54 ÷ 9 = 6	60 ÷ 10 = 6	66 ÷ 11 = 6	72 ÷ 12 = 6
63 ÷ 9 = 7	70 ÷ 10 = 7	77 ÷ 11 = 7	84 ÷ 12 = 7
72 ÷ 9 = 8	80 ÷ 10 = 8	88 ÷ 11 = 8	96 ÷ 12 = 8
81 ÷ 9 = 9	90 ÷ 10 = 9	99 ÷ 11 = 9	108 ÷ 12 = 9

EXERCISE 17.

Mental Exercises in Division.

1. How many 4's are in 12? in 16? in 48? in 24? in 36? in 28?
2. How many loads of 5 tons each are there in 40 tons? in 60 tons? in 35 tons? in 15 tons?
3. How many times can 7 be taken from 14? from 42? from 63? from 84?
4. Divide by 3, from 3 into 3 to 3 into 27.
   by 5, from 5 into 15 to 5 into 45.
   by 7, from 7 into 42 to 7 into 84.
   by 8, from 8 into 24 to 8 into 88.
   by 9, from 9 into 108 to 9 into 27.
   by 10, from 10 into 10 to 10 into 70.
   by 12, from 12 into 132 to 12 into 36.
5. What is the quotient in,

$45 \div 9$,	$36 \div 4$,	$72 \div 8$,	$56 \div 7$,	$108 \div 12$,	$77 \div 7$,
$81 \div 9$,	$72 \div 6$,	$54 \div 6$,	$132 \div 11$,	$64 \div 8$,	$30 \div 3$,
$16 \div 8$,	$42 \div 6$,	$80 \div 10$,	$27 \div 9$,	$35 \div 7$,	$28 \div 4$,
$16 \div 2$,	$14 \div 2$,	$60 \div 5$,	$35 \div 5$,	$84 \div 12$,	$63 \div 7$.

6. If a box holds 4 pounds of sugar, how many such boxes will be required to hold 36 pounds? 28 pounds? 16 pounds? 44 pounds?
7. 36 is how many times 9? 4? 6? 12?
8. 32 is how many times 4? 8?
9. 24 is how many times 2? 3? 4? 6? 8? 12?
10. 48 is how many times 4? 6? 8? 12?
11. From a pile of 60 bricks, how many loads of 12 bricks may be taken away?
12. If $56 be equally distributed among 7 men, how many dollars does each man receive?
13. When apples are 3 cents each, how many can I buy for 24 cents?

(In other words, how many times must three cents be repeated to give 24 cents,; the answer will be 8.

I could thus buy 8 apples. This must be correct, for each apple costing 3 cents, 8 apples must cost 8 times 3 cents, that is, 24 cents.)

14. If a man travel 6 miles an hour, how long will it take him to travel 54 miles?

15. How many tubs containing 9 gallons each can be filled from a hogshead containing 63 gallons?

16. If a man drive 8 miles an hour, in what time will he drive 56 miles?

17. A farmer bought some lambs for $60, paying $5 a head: how many lambs did he buy?

18. At $9 a week, in what time will a man earn $36? $54? $72? $81? $108?

19. If 7 barrels of sugar cost $63, what will 1 barrel cost?

20. If 6 kegs of powder cost $72, what will 1 keg cost?

21. If a man travel 48 miles in 4 days, how far does he travel in 1 day?

22. What will be the cost of 1 ton of coal, if 8 tons cost $64?

23. If you divide $84 among 7 children, how many dollars will each child have?

24. If a man build 72 feet of fencing in 8 days, how many feet can he build in 1 day?

25. If 9 dozens of fish cost 108 cents, what is the cost of 1 dozen?

26. How many articles at $12 each can be bought for $84? For $108? For $120?

27. How many lots of 5 acres each are in 20 acres?

28. How many barrels, each holding 3 bushels, will be required for 18 bushels of onions? For 21 bushels?

29. How many times can 6 yards of canvas be cut off from a piece containing 30 yards?

30. How many times can 6 cents be taken from 24 cents?

31. Distribute $28 equally among 7 people: how many dollars will each receive?

32. What is one of 4 equal parts of 40? Of 36? Of 48?
33. What is one of 6 equal parts of 30? Of 42? Of 48?
34. What is one of 7 equal parts of 56 pounds?
35. A teacher having 66 maps, distributed them equally in a class of 11 pupils: how many did each get?
36. If 96 pounds of bread are divided equally among 12 persons, how many pounds will each receive?
37. If 88 dollars are divided equally among 8 persons, how many dollars will each have?
38. If 120 barrels of flour are divided equally among 12 families, how much flour will each receive?
39. A master having 108 pupils, divided them into 9 equal classes: how many were in each class?
40. A picnic party of 11 persons spent $132: how much was that apiece?
41. A party of 10 persons found a purse containing $100, which they shared equally: how much did each receive?
42. A lad having $96, wishes to divide it equally among 8 friends: how much can he give to each?
43. If you pay 84 cents for a horse and waggon to go 7 miles, how much is that a mile?
44. A man having 120 feet of land, divided it into 6 equal lots; how many feet were there in each lot?

---

92. In all the previous examples of Division, the pupil must have noticed that the divisor was contained an exact number of times in the dividend. This is not always the case, for example:

I have 22 pears, and give 4 pears to each boy in the class: how many boys were there?

If there had been 5 boys, I would require only 20 pears; but if there had been 6 boys I must have 24 pears: so the only thing I can do is to give the 5 boys in the class 4 pears each, and keep the other 2 that remain for myself.

93. This number, 2 (in the case before us), is called the **Remainder**, and may be said to be that which is left after the divisor has been taken as many times as possible from the dividend.

If the remainder be first taken from the dividend, the result must contain the divisor exactly. Thus: 4 may be subtracted from, or contained in, 30, 7 times, but there will be 2 left, and we see that if the 2 be taken from the 30, the result, 28, will contain 4 exactly.

Ex. 1.—Divide 77 by 8.

Since 8 times 9 are 72, the quotient must be 9; and since 72 is less than 77 by 5, then 5 must be the remainder.

Ex. 2.—What must be taken from 49 that it may contain 9 **five times**?

The number that contains 9 **five times** we know to be 45, and since this number is 4 less than 49, 4 must be the required result.

EXERCISE 18.

☞ Mental Examples on the **Remainder.**

1. Give the quotient and remainder, if any, in—

18÷ 4,	21÷ 5,	62÷ 7,	41÷11,	39÷ 6,	71÷ 8,
90÷12,	80÷ 7,	23÷12,	62÷ 9,	73÷ 8,	45÷ 9,
83÷10,	31÷ 3,	42÷ 6,	70÷ 8,	75÷ 9,	120÷11,
140÷12,	93÷10.	48÷12,	79÷ 7,	80÷12,	100÷11.

2. What number must be divided by 6 to give a remainder 3 and quotient 5?

If 6 times 5, or 30, be divided by 6, the quotient will be 5 exactly, with no remainder. Hence, if there is to be a remainder of 3, the number must be 33.

PROOF.—6 is contained in 33, 5 times, and 3 over for a remainder.

3. What number must be divided by 5 to give—
   Quotient 7, remainder 2?
   Quotient 8, remainder 4?
   Quotient 11, remainder 3?

4. To give a quotient 7 and remainder 6, what number must be divided by 7? by 8? by 9? by 10? by 11? by 12?
5. A man had $100, and gave $9 apiece to 8 boys: how much had he left?
6. From 93 bushels of oats, how many horses can be allowed 10 bushels each, and what would be left?
7. A. receives what is left after dividing $100 among 8 men, and B. receives what is left after dividing $120 among 11 men: how much more does B. get than A.?
8. If I had 7 more apples, I could give 8 boys 11 apples each: how many have I?
9. Counting his marbles by sevens, Joseph had 11 lots and 4 over: how many had he?
10. How many could be put in each lot, to have the same number of marbles in each?
11. From Toronto to Hamilton is 46 miles; a man can ride 7 miles an hour: how far from Hamilton will he be at the end of 5 hours?
12. How many will be left over from 93 bank notes if they be tied in packages of 8? of 10? of 11? of 12?
13. There are 75 boys in the class, and 6 rows of seats: if 3 boys have to stand, how many seats are there in each row?
14. John has 47 plums, and gives 5 to each of his brothers and keeps the smallest share himself: how many brothers had John? what was his own share?
15. Four quarts of milk will fill a gallon measure: how many gallons would there be in a pail which holds 31 quarts? and how many quarts will be left after the gallon measures are filled?
16. If lamp-posts be placed 11 feet apart, and one placed in front of A.'s door, how many will there be between A.'s door and B.'s door, a distance of 139 feet? how far will the end one be from B.'s door?
17. Find the dividends, having—

Divisor	8,	Quotient	11,	and Remainder	3.
"	10,	"	4,	"	2.
"	12,	"	9.	"	7.
"	9,	"	8,	"	7.

18. Frank having 68 cents, bought 7 tops, and had 5 cents left: what were the tops apiece?

19. How many 3-cent postage stamps can you obtain for 35 cents?

20. How many 5-cent pieces can you obtain for 39 cents?

21. How many 8-dollar law stamps can be obtained for $67?

2. How many 8-cent loaves of bread can be made out of 75 cents' worth of flour?

23. How many times 7 in 8 times 8, and how many over?

24. How many times 9 in 7 times 8, and how many over?

25. In 7 times 9, how many times 6, and how many over?

26. In 8 times 11, how many times 9, and how many over?

27. In 9 times 12, how many times 11, and how many over?

---

94 All the previous examples have depended on a thorough knowledge of the Multiplication and Division Tables. No dividend has been larger than 144, and no divisor greater than 12.

The next case is that in which we have any dividend, but the divisor not greater than 12.

Ex. 1.—Divide 9639 by 3.

$$3)9639$$
$$\overline{3213}$$

The dividend is 9 thousands, 6 hundreds, 3 tens, and 9 units.

First, divide the 9 thousands by 3. The result or quotient is 3 thousands, which is written in its proper place under the dividend.

Then, 3 is divided into 6 hundreds, and gives 2 hundreds, which is placed, as usual, after the thousands.

Next, 3 tens divided by 3 gives 1 ten, and this is placed after the hundreds.

Finally, 9 units divided by 3 gives 3 units, which is written in the units' place.

The whole quotient is, therefore, 3 thousand 2 hundred and 13.

PROOF.—$3213 \times 3 = 9639$.

Ex. 2.—Divide 3955 by 4.

$$4)3955$$
$$988-3$$

The dividend is 3 thousands, 9 hundreds, 5 tens, and 5 units.

Since the first figure 3 does not contain 4, we say 3 thousands = 30 hundreds, making, with the 9 hundreds, 39 hundreds. Then 4 is contained in 39, 9 times and 3 over. That is, 9 hundreds, and 3 hundreds over. Place the 9 as usual, and carry the 3 hundreds on to the 5 tens, making 35 tens. Then 4 is contained in 35, 8 times and 3 over. That is, 8 tens and 3 tens over. Place the 8 tens after the 9 hundreds, and carry on the 3 tens, which with the 5 units make 35 units. Then, finally, 4 is contained in 35, 8 times and 3 over: that is, 8 units and 3 units over.

The quotient is 988, and the remainder 3.

The same method is used for all numbers, however large, in the dividend.

95. From this we have the following

RULE FOR DIVISION.

Write the divisor to the left of the dividend, drawing a line between them, also a line beneath the dividend.

If the divisor will exactly divide each figure in the dividend, place the quotients thus obtained in the proper order under the line. This will be the required quotient.

If the divisor will not exactly divide each figure in the dividend, proceed as follows:

Divide the divisor into the first figure, if possible; if not, into the first two figures; or, if not then, into the first three. Write this quotient in its own place under the dividend. If

there be a remainder, place it before the next figure in the dividend. Proceed as before, and carry the remainder to the next figure in the dividend, but if the divisor will not divide, write a nought below in the quotient, and carry the figure or figures to the next one in the dividend. Divide these as before, and so on till all the figures in the dividend are taken in. Place the remainder, if any, to the right of the quotient.

Ex.—Divide 116501 by 12.

12)116501

9708−5

Here 12 will not divide the first figure 1 or the two first figures 11, therefore we say 12 is contained in 116, 9 times and 8 over; put the 8 with the 5, and say 12 is contained in 85, 7 times and 1 over; take the 1 with the 0; but 12 is not contained in 10; therefore, put down a nought in the quotient, and take in the next figure 1 with the 10, and then say 12 is contained in 101, 8 times and 5 over. This 5 is the remainder, and it must be always less than the divisor.

Proof.—9708 × 12 = 116496
116496 + 5 = 116501

That is, multiply the quotient by the divisor, and to the result add the remainder, if any. This will give the dividend, if the work be correct.

96. Where the process is thus carried on mentally, and the quotient only set down, it is called **Short Division.**

EXERCISE 19.

Divide:

1. 624 by 2.	12. 9306 by 3.	23. 176 by 4.
2 862 by 2.	13. 746 by 2.	24. 215 by 5.
3. 684 by 2.	14. 368 by 2.	25. 252 by 6.
4. 396 by 3.	15. 459 by 3.	26. 364 by 7.
5. 693 by 3.	16. 756 by 3.	27. 434 by 7.
6 848 by 4.	17. 928 by 4.	28. 336 by 8.
7. 484 by 4.	18. 568 by 4.	29. 568 by 8.
8. 884 by 4.	19. 655 by 5.	30. 736 by 8.
9. 555 by 3.	20. 605 by 5.	31. 378 by 9.
10. 8642 by 2.	21. 9246 by 2.	32. 459 by 9.
11. 3693 by 3.	22. 136 by 4.	33. 8128 by 4.

Divide:

34. 6126 by 3.	41. 972 by 3.	48. 7535 by 5.
35. 5255 by 5.	42. 896 by 4.	49. 9372 by 2.
36. 6312 by 6.	43. 675 by 5.	50. 6185 by 5.
37. 8432 by 8.	44. 775 by 5.	51. 8491 by 7.
38 756 by 3.	45. 735 by 7.	52 9656 by 8.
39 978 by 2.	46. 8208 by 4.	53. 9981 by 9.
40. 872 by 4.	47. 6075 by 3.	54. 32568 by 3.

55. Twenty-seven thousand five hundred and twelve by eight.
56. Thirty-two thousand four hundred and ninety-six by six.
57. Fourteen million eight hundred and sixty-five thousand nine hundred and thirty-two by two.
58. Thirty-six thousand nine hundred and forty-five by nine.
59. Seventy-two thousand three hundred and forty-five by five.
60. Forty-five million eight hundred and twenty-eight thousand nine hundred and twenty-seven by nine.

☞ The pupil should prove the answers to each of the preceding questions, instead of referring to the answers in the book.

EXERCISE 20.

Find the quotient and remainder, if any, in each of the following questions, proving each result:

(1)	(2)	(3)	(4)
4)3654	5)72584	3)86471	7)40505
(5)	(6)	(7)	(8)
9)476589	12)987654	11)334523	8)639724

9. 2718065÷ 8.	19. 3706823÷ 9.	29. 7814873÷11.
10. 7893201÷ 9.	20. 6175802÷11.	30. 7076217÷12.
11. 5013487÷ 6.	21. 8160937÷12.	31. 1275923÷11.
12. 3920384÷ 7.	22. 5117284÷ 7.	32. 11330434÷12.
13. 8372146÷ 8.	23. 4465037÷ 8.	33. 41241154÷11.
14. 4365984÷ 9	24. 7600356÷ 9.	34. 2314205÷12.
15. 453678÷11.	25. 3978420÷ 7.	35. 3274604÷11.
16. 1496583÷12.	26. 4301765÷ 8.	36. 4677021÷12.
17. 5703214÷ 7.	27. 7400804÷ 9.	37. 8570198÷11.
18. 6183420÷ 8.	28. 4230569÷11.	38. 5891270÷12.

39. How many times may 3 be taken from 27021 ?

40. How often is 4 contained in 28032 ?

41. The divisor is 5, the dividend is 33515 : find the quotient.

42. How many 7's in 44268 ?

43. How many times must 6 be taken from 49392 to leave no remainder ?

44. How many 8's in 44248 ?

45. How many times 9 is 37845?

46. How many times 7 is 42924 ?

---

97. To divide by 10, 100, 1000, etc.

The number 3766 may be read 376 tens and 6 units. Thus we see that if one figure be cut off the right of a number, the remaining figures shew the number of tens there are in it : as 376 in the case above.

In the same manner, 3766 may be read 37 hundreds and 66. Thus, if two figures be cut off the right of a number, the remaining figures shew the number of hundreds in it : 37 in this case.

98. Therefore we see that to divide by 10, 100, 1000, 10000, etc., we need only cut off one, two, three, four, etc., figures from the right of the dividend, the quotient will be the remaining figures, and the figures cut off will be the remainder.

(Compare Art. 79.)

Ex.—Divide 87631 by 1000.

Cut off three figures from the right. The remaining figures, 87, will be the quotient, and the figures 631, that were cut off, will be the remainder.

99. This principle is very useful in the matter of dollars and cents, for, as there are 100 cents in every dollar, to bring any number of cents to dollars we need only cut off the last two figures as above, and the remaining number will be the required dollars, and the figures cut off will be the number of cents left over.

Ex.—86342 cents will be the same as 863 dollars and 42 cents; or,

86342 cents = $863.42,

The dot . being placed to separate the dollars from the cents.

(Compare Art. 81.)

EXERCISE 21.

Divide:

1. 7316 by 10.
2. 83174 by 10.
3. 6192 by 100.
4. 73001 by 1000.
5. 97312 by 10000.
6. 83916 by 100.
7. 513712 by 100.
8. 712934 by 100000.
9. 392 by 100.
10. 37214 by 1000.
11. 74321 by 1000.
12. 30600 by 100.
13. 3000000 by 100.
14. 6060600 by 100000.
15. Express 10862 cents in dollars, etc.
16. Express 312 cents in dollars, etc.
17. How many dollars will be the same sum as 461000 cents?
18. How many cents will be left if 861070 cents be exchanged for one-dollar bills?
19. How many dollar bills will be obtained?
20. How many dollars would buy as much land as 7310700 cents?

EXERCISE 22.

1. If 2 waggons of equal size carry 4896 bricks, how many bricks will one waggon carry?
2. If 2 houses are bought for 47054 dollars, how much is one of them worth?
3. If 3 mines cost 156378 dollars, how much does one mine cost?
4. If 3 times a certain price is $47901612, what is the price?

5. A grant of 60148 acres is to be divided among 4 persons: what is each one's share?

6. Divide $10632475 among 5 colleges: what will be the share of each?

7. Eight men have an equal interest in 268112 acres of land: how much has each?

8. If 9 square feet make one square yard, how many square yards are in 26002197 square feet?

9. In a market garden containing 8 acres there are 42336 hills of potatoes: how many hills are there in one acre?

10. Add the quotient of 36140292 divided by 9 to the quotient of 31623424 divided by 8

11. Divide 163207431 by 3 times 3.

12. A man died having an estate of 146329 dollars; his widow received 23193 dollars, and the remainder was divided equally among four hospitals: how much did each hospital receive?

13. I have 327 lemons, and sell 311: how many remain? how much shall I receive at 8 cents each for those I have sold?

14. At 2 cents each, how many apples can I buy for $43.44? The same money will buy how many toys, at 3 cents each? How many tarts, at 4 cents each?

15. At 2 dollars a day, how many men can I hire for 346 dollars? For 496 dollars? For 3176 dollars?

16. At 3 cents a spool, how many spools of thread can I buy for $3.84? For $5.73? For $49.62?

17. There are 4 pecks in a bushel: how many pecks are there in 3844 bushels? In 7688 bushels? In 15376 bushels?

18. There are 4 quarts in a gallon: how many gallons are there in 132 quarts? In 396 quarts? In 792 quarts?

19. How many pounds of sugar, at 9 cents a pound can I buy for $36.90? For $73.44?

20. At 4 dollars each, how many tickets can be bought for 64 dollars? For 192 dollars? For 1152 dollars?

21. There are 3 feet in 1 yard: how many feet are there in 27 yards? In 16 yards? In 29 yards?

22. Three feet make 1 yard: how many yards are there in 69 feet? In 276 feet? In 828 feet?

23. If 4 pumpkins weigh 108 pounds, how much will 1 of them weigh? If 3 weigh 108 pounds, what will 1 weigh?

24. If 4 iron rods of equal length measure 44 feet, what is the length of each? If they measure 132 feet, what is the length of each?

25. If 3 bushels of turnips will fill one barrel, how many barrels will 255 bushels fill?

26. A man bought a lot for 3792 dollars, which was 3 times as much as his house cost him: how much did his house cost him?

27. Four asylums are to share equally 7248 dollars: how much does each receive?

28. How many barrels of meal, at 5 dollars a barrel, can be bought for 3575 dollars?

29. At 4 dollars each, how many hats can be bought for 796 dollars?

30. At 11 dollars a barrel, how many barrels of vinegar can be bought for 1749 dollars?

31. There are 7 days in one week, how many weeks are in 365 days (one year)?

32. If 9 acres of land cost 1125 dollars, what will 1 acre cost?

33. If 6 cows cost 1272 dollars, what will 1 cow cost?

34. If a horse travels 693 miles in 7 days how far does he travel in 1 day?

35. 1704 acres of land are to be divided equally among 8 charities: how many acres will each receive?

36. If 9 mules sell for 1359 dollars, what will be the sum received for each?

37. A man bought 12 tons of hay for 180 dollars: how much did he pay a ton?

38. A boy sold 11 rabbits for $2.75: how much did he receive apiece?

39. A girl spent $3.54 for buttons, giving 3 cents apiece for them: how many did she buy?

40. X. is worth 15795 dollars, which is 5 times as much as Y. is worth, and Y. is worth 3 times as much as Z.: how much are Y. and Z. each worth?

41. A.'s land cost 2358 dollars, which is 3 times as much as the building of the house cost: what was the cost of the building?

42. A butcher bought 12 oxen for 1764 dollars: what was the average cost of each?

43. A cooper worked 12 months for 216 dollars: how much did he receive a month?

44. If 4 yards of tweed will make a coat, how many coats could be made out of 1876 yards?

45. A grocer spent 3661 dollars in sugar at 7 dollars per barrel: how many barrels did he buy?

46. How many barrels of cider at 5 dollars a barrel could be bought for 2235 dollars?

47. There are 4 weeks in 1 month: how many months are there in 5764 weeks?

48. A grocer spent $12.75 for baskets at 5 cents apiece: how many did he buy?

49. A school-house was built jointly by 7 gentlemen at an expense of 2625 dollars: what sum did each subscribe?

50. In 1 bushel there are 4 pecks: how many bushels are in 1176 pecks?

51. A mill worth 43652 dollars was owned by 7 men in equal shares: what was the value of a share?

52. If a train in 8 days runs 2896 miles, what would be the average run in 1 day?

53. A patent valued at 38125 dollars was owned in equal shares by 5 men: how much did each man own?

54. At 6 dollars a gallon, how many gallons of wine could be bought for 2274 dollars?

55. From the sun to the earth is about 92000000 miles; light travels this distance in about 8 minutes: how many miles does light travel in a minute?

100 We now come to those cases in Division, in which the dividend and divisor may be any numbers.

Ex.. 1.—Divide 18763 by 16.

16) 18763 (1000
16000

16) 2700 (100
1600

16) 1160 (70
1120

16) 43 (2
32

11 1172—11

The dividend is 18 thousand, 7 hundred, 6 tens, and 3 units.

16 is contained in 18 thousand 1 thousand times, leaving a remainder of 2 thousand, which with the 7 hundred make 27 hundred.

16 is contained in 27 hundred 1 hundred times, leaving a remainder of 11 hundred, which with the 6 tens make 116 tens.

16 is contained in 116 tens 7 times, leaving a remainder of 4 tens, which with the 3 units make 43 units.

16 is contained in 43 units 2 times, leaving a final remainder of 11 units.

The whole quotient is therefore 1 thousand, 1 hundred, 7 tens, and 2 units, or 1172, and the remainder 11.

The noughts to the right, expressing the thousands, hundreds, etc., are omitted in practice, because the place of the figures shews their value.

Ex. 2.—Divide 588491 by 83.

83) 588491 (7090—21
581

749
747

21

The least number of figures on the left of the dividend that will contain 83 is 5,8,8, that is, 588 thousand, and this contains 83, 7 times. 83×7 gives 581; or, in full, 83×7000 gives 581000. Subtracting the 581 from 588, we find the remainder to be 7. Bring down the next figure, 4, in the dividend, and we see that 74 will not contain 83; or, in other words, 74 hundreds will contain 83 no hundred times, and this no hundreds must be expressed in the quotient by the nought.

Since 74 will not contain 83, we bring down another figure, 9. 749 contains 83, 9 times, with a remainder 2.

Bring down the next figure, 1, and then, since 21 will contain 83 no times, we place a nought in the quotient and call the 21 our final remainder.

The following proof shews the correctness of the result:

83×7 thousands	=	581000
83×0 hundreds	=	0
83×9 tens	=	7470
83×0 units	=	0
		588470
Remainder . .		21
Dividend . . .		588491

101. When the divisor is greater than 12, and the different products are expressed, the process is called **Long Division.**

102. We then have the following

RULE FOR LONG DIVISION.

Write the divisor and dividend as before, leaving a place on the right for the quotient.

Find how many times the divisor is contained in the fewest number of figures on the left of the dividend. Place this as the first figure of the quotient.

Multiply the divisor by it, and subtract the product from these figures at the left of the dividend.

Attach, or bring down, to the difference, the next figure to the right in the dividend.

If the number thus formed will contain the divisor, place the number of times as the next figure in the quotient, and proceed as before; but if it does not contain the divisor, place a nought in the quotient, then bring down the next figure from the dividend.

Proceed in the same manner until all the figures in the dividend are brought down.

The number that is then left is the final remainder.

PROOF—*The same as in Short Division.*

☞ In finding the quotient figure, the pupil will be assisted by seeing how many times the first figure of the divisor is contained in the first figure, or, if necessary, the first two figures of the dividend; an allowance being made for the carrying figure.

☞ If any of the remainders (before bringing down a new figure) be **equal to** or **greater** than the divisor, it shows that the previous quotient figure is **too small**, and must be increased.

☞ If any of the products of the divisor by a quotient figure be **greater** than the number above it, it shows that the quotient figure is **too great**, and must be diminished.

☞ After the first quotient figure is obtained, there must be as many figures written in the quotient as there are figures brought down from the dividend.

EXERCISE 23.

Divide :

1.	588	by 28	14.	8995	by 35	27.	475524	by 612
2.	759	by 33	15.	9576	by 42	28.	1445204	by 802
3.	864	by 36	16.	9315	by 45	29.	1760225	by 905
4.	882	by 42	17.	27715	by 23	30.	3156584	by 722
5.	2996	by 14	18.	43692	by 33	31.	5173302	by 834
6.	3042	by 13	19.	82242	by 54	32.	5926431	by 643
7.	3995	by 17	20.	88641	by 63	33.	3214664	by 566
8.	5832	by 18	21	76875	by 75	34.	6923471	by 555
9.	5103	by 21	22.	35784	by 84	35.	14293624	by 675
10.	7524	by 22	23.	30618	by 126	36.	56243121	by 686
11.	5448	by 24	24.	38232	by 236	37.	692348726	by 897
12.	5668	by 26	25.	146448	by 324	38.	496839715	by 1047
13.	7099	by 31	26.	199864	by 301	39.	786935846	by 3118

40. How many times can $86 be taken from $17354?
41. $52 from $7012?
42. $17 from $13354?
43. $62 from $3406?
44. $73 from $45078?
45. $51 from $60702?
46. $55 from $13415?
47. How many 73's in 1731195?
48. " " 46's in 76131702?
49. " " 381's in 13261467?
50. " " 937's in 13189212?
51. " " 754's in 762294?
52. " " 112's in 51867?
53. " " 999's in 7281711?
54. " " 85's in 33490?
55. " " 556's in 3931476?
56. " " 2624's in 73484248?
57. " " 736's in 863256?

Find the quotients and also the remainders, if any, resulting from the following divisors and dividends:—

58. 3076 and 11214887.
59. 269181 and 1246038849.
60. 6739549 and 2331883954.
61. 2012 and 8659110.
62. 2309 and 78611003.
63. 3605 and 4843167.
64. 808 and 9863701.
65. 9101 and 4816657.
66. 7305 and 71810282755.
67. 6635 and 33216694340.
68. 7239 and 28956427101.
69. 3827 and 21998981374.
70. 5943 and 16518324782.
71. 73421 and 472698568233.
72. 85043 and 1172481547818.

73. Divide forty thousand two hundred and seventy-eight by seventy-five.
74. Divide seven hundred and sixty-five thonsand four hundred and thirty-one by ninety-six.
75. Divide three hundred thousand four hundred and twenty-eight by three hundred and twenty-four.
76. Divide forty-three million two hundred and ten thousand and forty by one thousand two hundred and thirty-six.
77. Divide fifty-six million thirty thousand one hundred and sixty-nine by two thousand and four.
78. Divide one hundred and nine million four hundred and twenty-six thousand and fifty-one by seven thousand three hundred and fifteen.
79. Divide four billion two hundred and eighty million nine hundred and sixty thousand three hundred and forty-two by fifteen thousand and three.
80. Divide thirty-one billion eighty-two million six hundred thousand five hundred and seventy-eight by four hundred and seven thousand and fifty-three.

---

103. It was shewn in Art. 83 that instead of multiplying by 16, for example, we could use its factors, 4 and 4 The same principle holds true in Division.

Ex. 1.—Divide 8413 by 24.

3) 8413

8) 2804 – 1

350 – 4

The factors of 24 are 3 and 8, those being the numbers which, when multiplied together, produce or make up 24. We therefore divide by 3, which gives 2804 and 1 over, and then divide the quotient by 8, which gives 350 and 4 over

Now, the 2804 represents that number of 3's; hence the 4 that was left on dividing it by 8 must mean four 3's, or 12.

This 12 with the 1 left over at first makes in all 13, which is the exact remainder.

PROOF: $350 \times 8 = 2800$
$2800 \times 3 = 8400$
$8400 + 13 = 8413$

Ex. 2.—Divide 42711 by 99.

$$9 \overline{)42711}$$
$$11 \overline{)4745-6}$$
$$431-4$$

$4 \times 9 = 36$, $36+6=42$ *Remainder.*

The factors of 99 are 9 and 11. The final quotient is 431.

The last remainder 4 means four 9's, or 36, and this with the first remainder 6 gives us the **true remainder**, 42.

104. We thus have the following

RULE FOR DIVIDING BY FACTORS.

Find the factors of the divisor. Divide the dividend, as usual, by one of them, and then this quotient by the other. This result will be the true quotient.

To find the true remainder, multiply the last remainder, if any, by the first divisor, and to the product add the first remainder, if any. The result will be the required remainder.

105. This principle enables us to divide more easily by any number ending in noughts: for example, 800. The factors of this number are 8 and 100, so we divide by the 100 first and then by the 8, and find the **true remainder** in the usual way.

Ex. 1.—Divide 97643 by 9000.

$$1000 \overline{)97643}$$
$$9 \overline{)97-643}$$
$$10-7$$

$7 \times 1000 + 643 = 7643$ *Remainder.*

106. This might have been done more rapidly thus

$$9 \overline{)97,643}$$
$$10-7643$$

For we can divide by the 1000 by merely cutting off the three figures to the right (Art. 79), then divide the remaining figures by the 9, and to the remainder, 7, attach the figures cut off, making in all 7643.

Ex. 2.—Divide 976341 by 3700.

```
37 ) 9763'41 ( 263
     74
     ---
     236
     222
     ---
      143
      111
      ---
       32          3241 Remainder.
```

First cut off the 41 to the right of the dividend, then to the remainder 32 attach the two figures 41, and we have the full remainder, 3241.

EXERCISE 24.

Divide:

1.	436899	by 14.	20.	8349	by 100.
2.	300527	by 18.	21.	7630	by 100.
3.	8307611	by 16.	22.	7491	by 1000.
4.	439205	by 21.	23.	$860000	by 100.
5.	4031729	by 24.	24.	312946	by 10000.
6.	843043	by 25.	25.	36972	by 10.
7.	7390478	by 28.	26.	131111	by 400.
8.	736255	by 42.	27.	23218	by 60.
9.	6310972	by 49.	28.	$22120	by 70.
10.	5084263	by 35.	29.	40220	by 1900.
11.	5083753	by 48.	30.	131127	by 12000.
12.	6230749	by 56.	31.	89952	by 500.
13.	4003767	by 36.	32.	7306597	by 30.
14.	5726009	by 44.	33.	4590000	by 306000.
15.	$19866	by 77.	34.	$13834500	by 120300.
16.	8514	by 99.	35.	11579112	by 890000.
17.	15336	by 72.	36.	3678900	by 326100.
18.	$93312	by 108.	37.	796532	by 230.
19.	4361	by 10.	38.	46512	by 8000.

How many dollars are there in

39. 4600 cents?
40. 1000000 cents?
41. 7200 cents?
42. 36000 cents?

Express as dollars and cents,

43. 846 cents.
44. 750062 cents.
45. 81243 cents.
46. 157 cents × 307

EXERCISE 25.

1. A man bought 15 horses for $1455: how much did he pay apiece?
2. A man paid $1400 for oxen: at the rate of $56 each, how many did he buy?
3. What will 1 barrel of oatmeal cost, if 37 barrels cost $296?
4. There are 28035 pages in 89 volumes: how many pages will be in each book?
5. A horse can travel 45 miles in a day: how long will he take to go 3330 miles?
6. The wages of Jones for 17 months come to $595: how much was he paid a month?
7. The cost of 97 sheep was $388: what did each cost?
8. If 95 lots of land cost $22515, what is that for one lot?
9. My agent sends me from Montreal 4368 hams, being 13 times too many: how many did I require?
10. If 15 boys work 5475 questions in 15 weeks, how many did each boy do?
11. There are 24 hours in the day: express 66360 hours in days.
12. In the workhouse there are 72 men whose ages amount to 5976 years: what is the average age of each?
13. I can employ 15 carpenters for $3555 for the season: what do I pay each man?
14. There is a new moon every 28 days: how many new moons will there be in 108192 days?

15. How many battalions can be formed out of 32340 soldiers, giving 420 men to each battalion?

16. How many 10-cent pieces would make up $64.60?

17. How many 25-cent pieces would pay a debt of $468?

18. A wealthy merchant distributed to 980 poor people, in an equal proportion, 876432 pounds of flour: what would each receive, and how much would be left?

19. If 63 gallons make a hogshead, how many hogsheads will there be in 1449 gallons?

20. How much would be left from $2449 after 116 men had been paid 21 dollars each?

21. An excursion boat can carry 105 people: how many trips must it run to take 2486 people, and how many go on the last trip?

22. The total outfit of a regiment of cavalry 1200 strong cost $236400: what was the cost of each man's outfit?

23. How many miles of road, at $26000 a mile, can be built for $11050000?

24. To give 236979, by what must I multiply 1809?

25. In an engagement, 4376 soldiers use 205672 cartridges: how many is that for each man?

26. How many feet are there in a mile, if 42 miles contain 221760 feet?

27. Of what number is 158 both divisor and quotient?

28. How many bales of cotton, each weighing 427 pounds, are there in a crop of 468419 pounds?

29. A moulder has 17385 pounds of metal: find the least number of pounds he must buy in order to cast cannon-balls each weighing 68 pounds?

30. How many could he then cast?

31. Divide one billion by 256.

32. The quotient is 345, the dividend is 273240: find the divisor.

EXAMPLES ON ALL PREVIOUS PRINCIPLES.

Ex. 1.—If 5 apples cost 10 cents, what must I pay for 8 apples?

If 5 apples cost 10 cents, 1 apple must cost 2 cents, and 8 apples would cost 8 times 2 cents, or 16 cents.

Ans. 16 cents.

Ex. 2.—How many pears at 3 cents apiece ought I to receive in exchange for 12 apples at 2 cents apiece?

In order to make the bargain even, the value of all the pears must be the same as the value of all the apples, which is 24 cents. How many pears, then, must there be to amount to 24 cents, at 3 cents apiece?

Ans. 8 pears.

☞ On account of the pears being worth more apiece than the apples, there must be a less number of pears than apples.

Ex. 3.—If 3 men can build a house in 21 days, how long must 7 men be employed to do the same work?

If 3 men take 21 days, one man would require 3 times as long as 3 men, that is, 63 days. One man doing the work in 63 days, 7 men would need only 9 days.

Ans. 9 days.

☞ The pupil must be taught very carefully to distinguish between Ex. 1 and Ex. 3. It is quite natural to reason thus: If 3 men take 21 days, 1 man would take 7 days; in fact, this is the very mistake the pupil will be apt to make.

Ex. 4.—A boy bought the same number of oranges as lemons, paying 5 cents each for oranges and 7 cents each for lemons: how many would he get for 84 cents?

If the boy had only 5 cents and 7 cents, that is 12 cents, he could only buy 1 orange and 1 lemon. Hence, for every 12 cents he owns he could buy one of each, and as he owns 84 cents, or 7 times 12 cents, he could buy 7 oranges and 7 lemons. *Ans.* 7 of each.

Ex. 5.—I gave 11 peaches to each of 8 boys, and kept 5 myself: how many had I at first?

For the boys alone I would need 8 times 11 peaches, or 88 peaches; but as I want 5 myself I must have 88+5, or 93, to begin with. *Ans.* 93 peaches.

In this problem the 93 is the dividend, the 8 is the divisor, 11 the quotient, and 5 the remainder. Hence we see that to find the dividend, 93, we multiply the divisor and quotient together, and add in the remainder.

Ex. 6.—A man buys a farm of 150 acres at 80 dollars an acre; he pays $5000 down, and the rest in 8 equal yearly payments: what does he pay each year?

The cost of the farm is $80 \times 150 = \$12000$. After paying $5000 down, there will be left $\$12000 - \$5000 = \$7000$ to be paid in 8 equal payments. Each payment must therefore be $\$7000 \div 8 = \$875$. *Ans.* $875.

EXERCISE 26.

1. A merchant left $4538 to his wife, $3289 to his daughter, and $2567 to his nephew: what was the amount of his property?
2. I bought a buggy for $475, and a pair of horses at $175 each: find the whole cost.
3. Which is worth more, 27 houses at $1125 each, or 205 oxen at $140 each: and how much?
4. Divide the product of 204 and 238, by their difference.
5. Seven hundred and eight men are formed in two rows: how many will be in each row? How many in each row if they were drawn up in four rows? How many if in six rows?
6. From what number must 72 be taken to leave a remainder equal to 3 times 45?
7. Find a number to which if 347 be added the sum will be 347 less than 1000.
8. How many times does a clock, if it sounds the hours, strike in a day?

9. A man bought 47 feet of land: for 25 feet he paid $41 a foot, for the rest $45 a foot: how much did all cost?

10. Add three hundred and sixty-two thousand four hundred and nine to eight hundred and seven thousand nine hundred and eighty-four, and divide their sum by eight.

11. A man left $14389 to be divided thus: to his widow $5000, to his son $4000, to each of four servants $100, and the rest to be equally divided among his three daughters: what will each of the daughters receive?

12. There are 24 sheets of paper in a quire: how many sheets in 3 dozen packets, each containing 5 quires?

13. A farmer bought 3 horses and 4 mules for $1122; the mules cost $144 each: what did each of the horses cost?

14. A merchant bought 13 bales of cloth, each bale containing 27 pieces, and each piece measuring 34 yards: what would be the value of the whole at 17 cents per yard?

15. If 36 men can cut a road in 77 days, how many men can do the same in 21 days?

16. How many yards of velvet at 7 dollars a yard, 8 dollars a yard, and 9 dollars a yard, the same quantity of each, can a dealer buy for 1800 dollars?

17. What number added to the product of 327 and 89 will give 30000?

18. When a man's property was divided, his son received $5148, and the rest was divided among 11 churches, giving each $936: what was the property worth?

19. Divide the sum of 5168 and 5206 by three times their difference.

20. How many weeks will it take a man to build 17 walls of 154 feet each, if he build 22 feet a week?

21. What must be multiplied by 327 to give 236421 gallons?

22. A. had 75 cows, B. 90 oxen; each sold his cattle for $2250: how much per head did A. receive more than B.?

23. I bought 16 pieces of print of 33 yards each at 10 cents a yard, and paid for them with tea at 80 cents a pound: how much tea was given?

24 The quotient is 345, the dividend 273240: what is the divisor?

25. The divisor is 213, the quotient 437, and the remainder 196: what is the dividend?

26. A. has 2280 dollars to lay out for horses and oxen, and wishes to purchase the same number of each: if he pays $65 a head for horses and $30 for oxen. how many of each can he buy?

27. I bought some books for $3.57, and sold them at 20 cents apiece, losing 17 cents: how many books were there?

28. A vessel sails 5712 miles in 48 days: how many miles does she go in a day? how many in 5 days?

20. A man's salary is $3150 a year; his expenses are $2817 a year: how much can he save in 6 years?

30. A.'s income is 5 times B.'s, B.'s income is 3 times C.'s, and C.'s income is $1325: find the incomes of all together.

31. How many cases, each containing 6 dozen books, can be filled from 18 parcels, each containing 3124?

32. If 27 clerks receive $3888 for 16 days' work, how much a day was that for each man?

33. What number is that to which if 17 be added the result is five times 384?

34. A man's income is 398 dollars a year: if he spend each year 256 dollars, how much will he save in 12 years?

35. A man had an income of $3742 a year (52 weeks); he spent $1500, gave to the hospital $370, and saved the rest: how much did he save per week?

36. If 563 be multiplied by a certain number, and 1043 be added, the result is 23000: find the number.

37. For 21 pigs and 43 calves a farmer received $401; the calves were sold at $2 each: what was the price of each pig?

38. Find a number, such that if the sum of 89 and 256 be subtracted from it, the remainder is 12 times 399.

39. A man bought wheat at 47 cents a bushel, and sold it for 56 cents a bushel; he gained $3276.45: how many bushels did he buy?

40. What number must be multiplied by 37 to make the product equal to the sum of 1998 and 3996?

41. Eight pounds of tea cost $4.80: what was the price of 6 pounds?

42. If 11 men can sod an acre of ground in 12 days, how many days will 4 men take to do the same amount of sodding?

43. If 144 dollars would pay for 24 yards of cloth, what will be the price of 56 yards of the same kind?

44. Seven sheep can be bought for $24.50: how much will a flock of 133 cost?

45. If 4 bushels of apples can be bought for $3, how many, at the same rate, can be bought for $51?

46. If 6 oranges can be bought for 42 cents, how many will 56 cents pay for?

47. In how many hours can 2 boys do as much work as 6 boys in 3 hours?

48. In how many days can 4 boys earn as much as 8 boys in 6 days?

49. In how many weeks can 15 men earn as much as 3 men in 25 weeks?

50. In how many days will 6 horses eat as much as 18 horses in 5 days?

51. Divide the sum of 1692 and 1786 by their difference.

52. A man exchanged 159 coats at $5 each for a horse valued at $144, and the balance in hats at $3 apiece: how many hats did he receive?

53. A. traded with B., giving 305 paintings at $45 each, and receiving 77 reapers at $181 each: which owes the other, and how much?

54. $89648 is 8 times as much as I paid for a house; but I paid $126 more than the house was worth: what was it worth?

55. I bought 312 barrels of oatmeal at $5 a barrel, and sold them for $2496: how much was gained on each barrel?

56. By selling 31 lots for $3100 I lose $155: for what should I sell 16 lots to gain $597?

57. A man's yearly salary was $9237; he spent $136 on house repairs; for hired men he paid 4 times as much, lacking $95; and for other expenses, $1902: what has he left to put by yearly?

58. I bought 25 sacks of flour for $125: what must I sell them for per sack to gain $75?

59. What will be the gain on each sack in the last question?

60. Jones sold the same number of plover, snipe, and quail for $17.70—the plover at 12 cents, the snipe at 37 cents, and the quail at 69 cents each: how many of each did he sell?

61. Ten thousand railway checks are to be marked by 3 men; the first marks 200 an hour; the second and third each mark 150 an hour: how long will they take to mark the whole, all working together?

62. If 59 articles cost me $43.07, how much must I sell 23 of them for to gain $1.83 on those sold?

63. A man earns $50 a month, but it costs him $30 a month to live: how many months will he take to save enough to purchase 48 acres of land at $10 an acre?

64. I sold 28 horses at $122 each; then bought 224 sheep at $12 each, 8 cows at $60 each, and spent the remainder in calves at $8 each: how many calves did I buy?

65. If it costs $56 for bricks to build a cistern, when bricks are worth $8 a thousand, what will it cost for bricks to build it, when they are worth $10 a thousand?

66. If 5 barrels of cider are worth $20, how many tons of hay, at $12 a ton, will 9 barrels of cider buy?

67. A merchant bought 3 pieces of cloth of equal lengths at $5 a yard; he gained $35 on the whole cost by selling 2 pieces for $350. How many yards in each piece?

107. The following examples will be found somewhat similar to those in Exercise 16, and although rather more difficult, the pupil should now be able to solve them mentally with a fair degree of rapidity.

The note in Art. 85 applies equally well to this case.

EXERCISE 27.

1. To 5 add 7, multiply by 3, subtract 6, divide by 5, multiply by 8, divide by 4, add 8: what is the result?
2. From 15 take 8, multiply by 6, divide by 7, add 10, divide by 8, add 20, subtract 4, divide by 3, multiply by 7: what is the result?
3. Multiply 7 by 8, subtract 2, divide by 6, add 7, divide by 4, add 26, divide by 5, multiply by 7, add 6, divide by 8, multiply by 4: result?
4. Divide 45 by 5, multiply by 3, add 8, divide by 7, add 31, subtract 4, divide by 8, multiply by 9, add 6, divide by 7, add 15, subtract 9, multiply by 8: result?
5. Add 9 to 19, divide by 7, multiply by 8, take 7, divide by 5, multiply by 12, subtract 4, divide by 8, add 27: result?
6. Subtract 7 from 25, divide by 6, multiply by 9, add 8, divide by 7, multiply by 20, subtract 4, divide by 12, multiply by 3: result?
7. To the product of 7 and 5, add 9, divide by 11, multiply by 12, subtract 3, divide by 9, multiply by 10, add 6, divide by 7, add 8, divide by 2, add 19, divide by 9, multiply by 12: result?
8. To the quotient of 63 divided by 7, add 6, multiply by 4, divide by 12, add 30, divide by 7, add 16, divide by 7, multiply by 11, add 9, divide by 7, add 15, subtract 7, divide by 7, add 16: result?
9. To the difference between 7 and 15, add 10 divide by 6, multiply by 11, add 9, divide by 7, multiply by 9, subtract 6, divide by 8, multiply by 7, add 9: result?

10. To 23 add 9, divide by 8, add 35, divide by 3, add 8, divide by 7, multiply by 20, divide by 10, add 27, subtract 9, divide by 8, multiply by 15, add 19 : result ?

11. From 41 take 5, divide by 9, multiply by 11, add 12, divide by 8, multiply by 7, subtract 5, divide by 4, multiply by 7, subtract 5, divide by 9, add 34, divide by 2 : result ?

12. Add 35 to 9, divide by 11, multiply by 25, subtract 16, divide by 12, add 43, divide by 5, add 53, subtract 13, divide by 5 : result ?

13. Multiply 7 by 8, add 10, divide by 11, add 21, divide by 9, multiply by 12, add 12, divide by 8, multiply by 11, divide by 22 : result ?

14. Divide 72 by 9, multiply by 7, subtract 8, divide by 6, multiply by 12, add 12, divide by 9, add 52, divide by 8, multiply by 10, divide by 20 : result ?

15. To 61 add 11 divide by 6, subtract 11, add 55 divide by 7, multiply by 6, subtract 18, divide by 6, multiply by 30, take 50, divide by 5 : result ?

16. From 85 take 15, divide by 7, multiply by 8, add 16, divide by 8, add 30, divide by 7, multiply by 12, add 8, divide by 8 : result ?

17. Multiply 30 by 4, divide by 12, add 25, divide by 7, multiply by 11, add 9, divide by 8, multiply by 5, subtract 7, add 3, divide by 9 : result ?

18. Add 17 to 20, subtract 9, divide by 7, multiply by 25, subtract 4, divide by 8, add 8, multiply by 4, subtract 20, add 30, divide by 10, multiply by 8, add 12, divide by 12, add 10 : result ?

19. Divide 56 by 8, multiply by 9, divide by 7, multiply by 8, add 5, divide by 11, multiply by 6, add 21, divide by 9, add 20, divide by 3, multiply by 5, add 15, divide by 10, multiply by 8, divide by 6 : result ?

20. From 63 take 9, add 16, divide by 10, add 41, subtract 20, divide by 4, add 93, subtract 17, add 2, divide by 5, multiply by 3, subtract 8, add 27, divide by 7, subtract 10, multiply by 13 : result ?

CHAPTER II.

FACTORING.

108. There are two ways of making up the number 6, either by adding 4 and 2, or by multiplying 3 and 2.

As in Art. 83, where the number 6, is made up by multiplying 3 and 2, each of these numbers is called a **Factor** of 6. Each of them is also an **Exact Divisor** of 6.

109. A **Factor** of a number may therefore be said to be an **Exact Divisor** of the number.

It is very desirable that the pupil should be able to tell the different Divisors of any number.

Ex. 1.—Find all the Divisors of 18.

$$18 = 9 \times 2, \text{ or } 3 \times 6,$$

Hence the Divisors are 9, 6, 3, 2.

110. The numbers 2, 3, 5, 7, 11, etc., have no exact Divisors or Factors, and all such numbers are called **Prime Numbers.**

Since the number 3 divides both 6 and 9, 3 is said to be a **Common Divisor** of 6 and 9. So 5 is a Common Divisor of 15 and 20; 4 is a Common Divisor of 8, 12, and 16.

111. A **Common Divisor** is any number that will exactly divide two or more numbers.

Ex.—Find a Common Divisor of 16, 20, 24.

$$16 = 2 \times 8.$$
$$20 = 2 \times 10.$$
$$24 = 2 \times 12,$$

Therefore 2 is a Common Divisor.

Again, ... $16=4\times4$,
$20=4\times5$,
$24=4\times6$,

Therefore 4 is also a Common Divisor.

We thus see that there may be more than one Common Divisor to two or more numbers; and, since 4 is the greater of the two Divisors, it is called the **Greatest Common Divisor** of 16, 20, and 24.

112. The **Greatest Common Divisor** (G.C.D.) is the greatest number that will exactly divide two or more numbers.

Ex.—Find the Greatest Com. Div. of 18, 24, 30.

We see, by inspection, that 2, 3, and 6 are the only Common Divisors of 18, 24, and 30, therefore the G. C. D. is 6.

☞ The Divisors, Common Divisors, and Greatest Common Divisors should, if possible, be found by inspection.

113. The following will be found a simple method of finding the G. C. D. of any numbers:

Take 16, 24, and 50.

The G. C. D. cannot be greater than 16, and must be some divisor of 16. The greatest divisor of 16 is 8, but this will not divide 50. The next divisor of 16 is 4, but this will not divide 50. The next divisor is 2, and since this also divides 24 and 50, it must be the G. C. D.

114. This gives us the following

RULE FOR FINDING THE GREATEST COMMON DIVISOR.

Take the least of the given numbers and try its divisors in order, beginning with the greatest.

The first one that will divide each of the other numbers will be the required Greatest Common Divisor.

Ex.—20, 24, and 28.

The divisors of 20 are 10, 5, 4, and 2. The first one that divides both 24 and 28 is 4. Hence 4 is their G. C. D.

EXERCISE 28.

☞ These questions should be solved mentally.

1. What numbers will exactly divide 12? 48? 56? 81?
2. Find the exact divisors of 21; 32; 49; 42; 36.
3. What numbers under 50 are exactly divisible by 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?
4. What numbers between 50 and 121 have for a factor 5? 7? 9? 12? Write down the other factor in each case.
5. Write down the simplest or prime factors of 64, 54, 78, 120, 145, 152, 99, 117, 189.
6. What numbers less than 150 are divisible by both 3 and 4? 4 and 5? 5 and 6? 3 and 8?
7. Name the three least numbers that exactly contain both 3 and 5; 2 and 5; 2 and 3; 3 and 4; 4 and 5.
8. Write down in order the prime numbers less than 50; between 50 and 100.
9. What three prime numbers will divide 42? 30? 105?
10. Find the common divisors of 24 and 30; of 27 and 36; of 15 and 45; of 36 and 64; of 72 and 80; of 90 and 120.
11. Name all the common divisors of 12, 18 and 20; of 24, 40 and 60; of 36, 48 and 72; of 24, 36, 60, 72.
12. Write down the G. C. D. in each part of questions 10 and 11.
13. What is the G. C. D. of 16, 24, and 36? of 9, 27, and 33? of 15, 35, and 50? of 18, 32, and 60?

115. When the given numbers are large, the G. C. D. can not always be found by inspection.

The following method is then adopted :

Ex.—Find the G. C. D. of 697 and 820.

```
697)820(1
    697
    ---
123)697(5
    615
    ---
 82)123(1
    82
    --
 41)82(2
    82
    --
```

Divide the less into the greater number—the remainder is 123, which we divide into the first divisor 697. This leaves a remainder 82, which we divide into the previous divisor 123, leaving a remainder 41. The number 41 is divided into the previous divisor 82, and since it is contained exactly, 41 is the **G. C. D.**

☞ In finding the G. C. D., if the last divisor is 1, the given numbers are said to be prime to one another.

116. From the above we have the following

RULE FOR FINDING THE GREATEST COMMON DIVISOR.

Divide the less into the greater of the given numbers, then divide the remainder then obtained into the previous divisor, and so on, until an exact divisor is obtained. This exact divisor will be the G. C. D. required.

☞ If there be three or more numbers, find the G. C. D. of any two of them. Then find the G. C. D. of this result and a third number and so on. The final result will be the G. C. D. required.

Ex.—Find the G. C. D. of 585, 765, and 285.

The G. C. D. of 585 and 765 is 45. The G. C. D. of 45 and the third number 285 is 15. Hence 15 is the G. C. D. of the given numbers.

☞ The pupil should prove the truth of the result, in other words, see that the G. C. D. obtained will exactly divide each of the given numbers.

Thus: $585 \div 15 = 39$.
$765 \div 15 = 51$.
$285 \div 15 = 19$.

EXERCISE 29.

1. Find the greatest common divisor of 161 and 115.
2. Find the greatest common divisor of 592 and 332.
3. Find the greatest common divisor of 2013 and 1220.
4. Find the greatest common divisor of 576 and 960.
5. Find the greatest common divisor of 592 and 1225.
6. Find the greatest common divisor of 1369 and 703.
7. Find the greatest common divisor of 1866 and 1492.
8. Find the greatest common divisor of 1029 and 1197.
9. Find the greatest common divisor of 992, 352 and 672.
10. Find the greatest common divisor of 867, 1088 and 714.
11. Find the greatest common divisor of 1134, 1386 and 630.
12. What is the length of the longest pole that will measure 84 feet, 56 feet and 70 feet?

---

117. Wherever we have a **Divisor** we must have a **Dividend.** The object of the previous exercise was to find the **Divisor.** We shall now proceed to find the **Dividend**, of which certain numbers are given as **Divisors.** When we speak of a **Divisor** or a **Dividend,** we always refer to an **Exact Divisor** and an **Exact Dividend.**

Since $3 \times 4 = 12$, 12 contains both 3 and 4, and is therefore an **Exact Dividend** of 3, and also of 4.

118. An **Exact Dividend** of a given number is therefore a number which will contain the given number without any remainder.

An **Exact Dividend** is also called a **Multiple.**

Since 15 contains 3 and also 5 exactly, 15 is a dividend of 3 and also of 5, and is called a **Common Dividend** of 3 and 5.

119. A **Common Dividend** is a number that contains two or more numbers exactly.

Ex.—24 is a Common Dividend of 2 and 4. 18 is a Common Dividend of 2, 3, and 6.

Now, 36 is a Common Dividend of 2, 3, and 4; and so likewise is 24, and also 12. And since of the common dividends 36, 24, and 12, 12 is the least, it is called the **Least Common Dividend** of 2, 3, and 4.

120. The **Least Common Dividend** (L. C. D.) of two or more numbers is the least dividend that will contain each of the numbers exactly.

Ex.—The L. C. D. of 2, 3, and 8 is 24, because 24 is the least dividend that will contain 2, 3, or 8. The L. C. D. of 4, 5, 12 is 60.

☞ All Dividends, Common Dividends, and Least Common Dividends should be found, if possible, by inspection.

121. The following will be found a good method of finding the L. C. D. mentally:

Take 5, 8, and 12.

The L. C. D. cannot be 12, because 12 does not contain either 5 or 8. The next number that contains 12 is 24, but this does not contain 5, although it contains 8. Then we try 3 times 12, 4 times 12, etc., until we come to 9 times 12, or 108. None of these will answer, but the next one, 10 times 12, or 120, contains both 5, 8, and 12, and must be the L. C. D.

122. Hence we have the following

RULE FOR FINDING THE LEAST COMMON DIVIDEND.

Take the greatest of the given numbers, and try its dividends in order, beginning with the least. The first one that will contain each of the other numbers will be the required Least Common Dividend.

Ex.—Find the L. C. D. of 10, 24, and 30.

The successive dividends of 30 are 60, 90, and 120; and since 120 is the first one that will contain 10 and 24, the L. C. D. must be 120.

EXERCISE 30.

☞ These questions should be solved mentally.

1. What numbers below 50 are dividends of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12?

3. What numbers between 50 and 145 exactly contain 7? 9? 11? 12?

4. Of what two prime numbers is 12 a common dividend?

5. Of what numbers is 12 a common dividend? 18? 24? 30?

6. Write in order each number below 100 that is a common dividend of 2 and 3; of 3 and 7; of 2, 3, and 4; of 4, 5, and 6; of 2, 5, and 7; of 2, 3, 4, and 5; of 3 and 8; of 8 and 10.

7. Write down the L. C. D. in each part of question 6.

8. Find the L. C. D. of 8 and 12; of 9 and 12; of 2, 4, 5, and 6; of 2, 3, and 10; of 3, 4, and 8; of 4, 5, and 8; of 6, 3, and 8; of 2, 5, 8, and 10; of 3, 12, and 4; of 6, 18, and 9; of 4, 12, and 16; of 8, 10, and 12.

123. In finding a common dividend of 3, 4, 12, 15, we need not consider 3 or 4, because any number that is a dividend of 15 must be a dividend of 3; and any number that is a dividend of 12 must be a dividend of 4.

Hence in finding the L. C. D. of any numbers we may always strike out any one of them that is a divisor of any other.

124. We thus have the following

RULE FOR FINDING THE LEAST COMMON DIVIDEND OF SEVERAL NUMBERS.

Place the given numbers in a line, and first strike out any one of them that is a divisor of any other.

Then begin with the lowest divisor, 2, and divide by it as often as it is contained in any two of the numbers, bringing down any numbers that are not divisible.

Proceed thus with 3, 5, 7, etc., always striking out in any line any number that is a divisor of any other number in that line.

Finally, multiply together the different divisors and all the numbers in the last line.

The product thus obtained will be the required L. C. D.

Ex.—Find the Least Common Dividend of 4, 6, 10, 12, 30, 45, 75, 100.

2)	4,	6,	10,	12,	30,	45,	75,	100
2)				6,	15,	45,	75,	50
3)				3,		45,	75,	25
5)						15,	25,	
						3,	5,	

L.C.D. $= 2 \times 2 \times 3 \times 5 \times 3 \times 5 = 900$. *Ans.*

Writing the numbers in a line, we first strike out 4 and 6, since each is a divisor of 12. Then strike out 10, since it is a divisor of 30. (Art. 123.)

Now divide by 2, and we obtain the quotients as above, of which we strike out 15, because it is contained in 45.

Again divide by 2, and then strike out 3 and 25, for each is contained in 75.

Now, since 2 will no longer divide any two numbers in the line, we try 3, and then 5.

Finally, multiply together the four divisors and the last quotients, and we obtain the L. C. D. as above.

125. To find the L. C. D. of two large numbers, we first find the Greatest Common Divisor.

Then divide this G. C. D. into either of the numbers, and multiply the quotient by the other number.

The product will be the required least common dividend.

Ex.—Find the L. C. D. of 970 and 1261.

Their greatest common divisor will be found to be 97.

Then $970 \div 97 = 10$.

$1261 \times 10 = 12610$, which is the required L.C.D.

Proof:

$12610 \div 970 = 13$.

$12610 \div 1261 = 10$.

126. If there be several large numbers, find the L. C. D. of any two, then find the L. C. D. of this result and a third number, and so on.

The final result will be the required L. C. D.

☞ As in the case of the Greatest Common Divisor, the pupil should prove the truth of the result by finding, as in Art. 125, if the L. C. D. will contain each of the given numbers exactly.

EXERCISE 31.

Find the Least Common Dividend:

1. Of 5, 15, 9, 6 and 3.
2. Of 4, 5, 10, 8, 18 and 15.
3. Of 12, 36, 25, 60, 35 and 72.
4. Of 63, 81, 14, 54, 27 and 9.
5. Of 7, 72, 84, 42, 12 and 6.
6. Of 72, 36, 180, 24, 18, 9 and 120.
7. Of 90, 10, 64, 70, 45, 8 and 32.
8. Of 40. 24, 8, 32, 20, 16 and 10.
9. Of 29, 144, 216, 180, 90 and 252.
10. Of 60, 78, 42, 96, 56, 48 and 39.
11. Of 2041 and 8476.
12. Of 812 and 336.
13. Of 7056 and 7392.
14. Of 7212, 9015 and 24040.
15. Of 7218, 6015, and 5213.
16. Of 2712, 816, 54, and 15.
17. Of 250, 360, 49, and 700.
18. Of 32, 44, 52, 13, 65, and 48.
19. Of 76, 748, 448, 152, and 38.
20. What is the smallest quantity of barley that can be carted away in either 20, 25, 30, 35, or 40 bushel carts, and how many loads would there be of each?
21. Find the least amount of money that can be paid by either 2, 3, 4, 5, 10, 20, 50, or 100 dollar bills and how many of each kind would be required?

CHAPTER III.

FRACTIONS.

127. We have spoken of **one** dollar, **one** pound, **one** pint, **one** foot, etc., but have not mentioned any smaller part than **one** of each. We must now see how we can express any part of a dollar, a pound, a pint, or a foot.

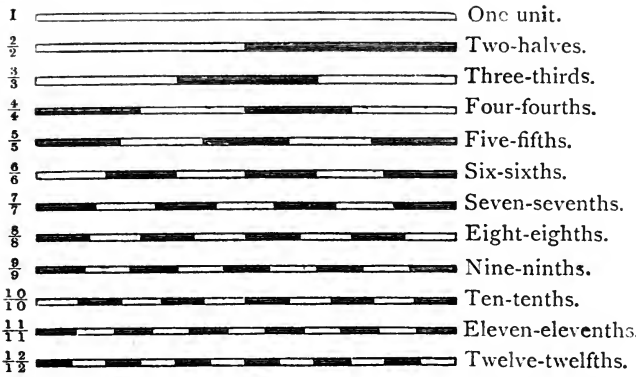
If we divide a **foot**, for instance, into two equal parts, each part is called **a half**, and written $\frac{1}{2}$—the figure 2 showing that the unit (in this case **a foot**) is divided into **two** equal parts, and the figure 1 showing that we have taken **one** of these parts.

In **the** same way, if a **foot** be divided into 3, 4, 5, 6, etc., equal parts, each part will be called a **third**, **fourth**, **fifth**, **sixth**, etc., respectively, and be represented by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc.

It will also be seen that to make up the whole, or **the** unit, we must take

Two-halves, or
Three-thirds, or
Four-fourths, or
Five-fifths.
etc., etc.

This will be easily seen from the following figure, where the unit is taken as a **foot** in length.



We have taken the unit to be one foot, and since there are twelve inches in a foot, each part of the lower line must represent an inch in length.

128. The following result will then be seen by using a straight-edge or measure :

The whole	equals	12 inches.
One-half	"	6 inches.
One-third	"	4 inches.
One-fourth	"	3 inches.
One-sixth	"	2 inches.

Thus we see that we find one-half of any number by dividing it by 2, one-third by dividing it by 3, one-fourth by 4, etc.

Hence, $\frac{1}{4}$ of 40 cents = 10 cents.
$\frac{1}{9}$ of 81 apples = 9 apples.
$\frac{1}{15}$ of 30 days = 2 days.

129. Again, referring to our figure, we see that two-thirds of 12 inches are 8 inches, that is, twice as long as one-third ; three-sixths are 6 inches, or three times as much as one-sixth ; seven-twelfths are 7 inches, or 7 times as much as one-twelfth.

In the same manner, $\frac{5}{11}$ of \$44 must be 5 times $\frac{1}{11}$ of \$44, that is 5 times \$4, which is equal to \$20.

So, $\frac{4}{9}$ of 63 = 4 times $\frac{1}{9}$ of 63 = 28.
$\frac{10}{13}$ of 26 = 10 times $\frac{1}{13}$ of 26 = 20.

130. The symbols $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{5}$, etc., are called **Fractions**, and represent one or more of the **equal parts** of the whole or unit.

131. The lower number is called the **Denominator**, because it points out or shows the number of equal parts into which the unit is divided, or, in other words, **it shows the size of the parts.**

132. The upper number is called the **Numerator, for it tells the number of parts taken.**

Thus $\frac{9}{11}$ is a Fraction, and represents nine of the equal parts of the unit. 11 is the Denominator, and shows the size of the parts to be elevenths. 9 is the Numerator, for it tells that the number of parts taken is nine.

EXERCISE 32.

☞ These questions should be solved mentally.

1. Read the following fractions, naming the denominator and numerator to each: $\frac{3}{8}$, $\frac{1}{7}$, $\frac{4}{5}$, $\frac{5}{11}$, $\frac{7}{15}$, $\frac{4}{9}$, $\frac{13}{15}$, $\frac{4}{15}$, $\frac{7}{18}$, $\frac{6}{11}$, $\frac{8}{9}$, $\frac{15}{16}$, $\frac{7}{15}$, $\frac{8}{20}$, $\frac{7}{16}$, $\frac{1}{10}$.

2. Write the following fractions in figures:

Five-ninths.	Five-elevenths.
Three-sevenths.	Nine-fourteenths.
Two-fifths.	Eleven-twelfths.
Four-sevenths.	Eight-fifteenths.
Five-sixths.	Seven-twentieths.
Three-eighths.	Eight-thirteenths.
Four-ninths.	Four-twentieths.
Seven-ninths.	Eleven-nineteenths.
Seven-tenths.	Sixteen-twenty-thirds.
Five-sevenths.	Three-fourteenths.
Eight-ninths.	Four-fortieths.
Nine-tenths.	One-seventieth.
One-twelfth.	Seventy-ninetieths.

3. When anything is divided into seven equal parts, what is one part called? Three parts? Five parts?

4. What is one of the eleven equal parts of anything called? Seven of the twelve equal parts? Nine of the ten equal parts? Fourteen of the fifteen equal parts? Eighteen of the twenty equal parts?

5. What is meant by one-ninth of a quantity of apples? Seven-elevenths of a heap of oats? Ten-twelfths of a distance? Four-twentieths of the value of a vessel? Five-sevenths of a man's property?

6. How many sixths in the whole of an estate? Tenths in one-half of an apple? Quarters in one-half of a yard? Sixths in one-third of a pound? Eighths in one-quarter? Sixteenths in one-eighth?

7. What is $\frac{1}{2}$ of 8? $\frac{1}{3}$ of 12? $\frac{1}{5}$ of 25? $\frac{1}{7}$ of 63? $\frac{1}{9}$ of 18? $\frac{1}{11}$ of 55? $\frac{1}{12}$ of 48? $\frac{1}{10}$ of 120? $\frac{2}{3}$ of 24? $\frac{3}{5}$ of 45? $\frac{4}{11}$ of 88? $\frac{3}{7}$ of 21? $\frac{4}{13}$ of 39? $\frac{2}{3}$ of 27 pounds? $\frac{3}{4}$ of 48 ounces? $\frac{4}{5}$ of 1 ounce, or 20 pennyweights? $\frac{5}{6}$ of 42 inches? $\frac{8}{11}$ of 77 acres? $\frac{10}{11}$ of 66 yards? $\frac{5}{7}$ of \$84?

8. How much of anything will be left if $\frac{1}{7}$ be taken away? If $\frac{3}{10}$ be taken away? If $\frac{3}{7}$ be taken away? If $\frac{5}{12}$ be taken away? If $\frac{2}{11}$ of it be lost? If $\frac{4}{9}$ of it be given away? If $\frac{9}{16}$ of it be sold? If $\frac{10}{17}$ of it be lost?

9. How much of anything must be taken from it to leave $\frac{5}{6}$ of it? $\frac{3}{10}$ of it? $\frac{5}{12}$ of it? $\frac{9}{13}$ of it? $\frac{4}{7}$ of it? $\frac{12}{17}$ of it? $\frac{4}{15}$ of it? $\frac{11}{12}$ of it?

10. What part of my farm may I sell to have $\frac{4}{5}$ of it left? $\frac{3}{7}$ of it? $\frac{5}{11}$ of it? $\frac{12}{19}$ of it? $\frac{3}{11}$ of it? $\frac{5}{12}$ of it? $\frac{3}{16}$ of it? $\frac{1}{8}$ of it?

11. A farm contains 260 acres: how many acres in $\frac{4}{5}$ of it? in $\frac{3}{10}$ of it? in $\frac{3}{13}$ of it? $\frac{1}{4}$ of it?

12. If $\frac{3}{4}$ of a vessel be worth \$60, what will $\frac{1}{4}$ be worth?

13. If $\frac{5}{6}$ of my property be 100 acres, find the number of acres in $\frac{1}{6}$ of it.

14. If $\frac{2}{3}$ of a number be 16, what will $\frac{1}{3}$ of it be?

15. If $\frac{3}{4}$ of a bushel of oats be worth 72 cents, what would $\frac{1}{4}$ be worth? What would a bushel be worth? What would $\frac{1}{12}$ of a bushel be worth? $\frac{6}{12}$ of a bushel? $\frac{7}{13}$ of a bushel?

16. How old is a boy $\frac{3}{7}$ of whose age is just 6 years?

17. If $\frac{4}{5}$ of a pound of tea be worth 72 cents, find the price of a pound.

18. What is coffee worth a pound when 40 cents pays for $\frac{2}{3}$ of a pound?

19. I bought a keg of nails: having but $8 I could only pay for $\frac{4}{5}$ of it. What must I yet pay?

---

133. We have seen that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{6}{12}$ of the unit, in Art. 127, was the same in each case, namely, 6 of the 12 equal parts or inches. This shows that a fraction may be expressed in many different forms, and still be the same in value. In other words, if we decrease the size of the parts we must take more of them, and if we increase the size of the parts we must take fewer of them. Again, if we increase the number of parts taken we must decrease their size, and if we decrease the number of parts taken we must increase their size.

The Numerator and Denominator are called the Terms of a Fraction.

Since $\frac{1}{2}=\frac{2}{4}$, $\frac{2}{4}=\frac{6}{12}$, etc., we deduce the following important Principle:

134. *If the two terms of any fraction be multiplied by the same number, the value of the fraction is not changed.*

Ex.—$\frac{3}{5}=\frac{15}{25}$; $\frac{2}{3}=\frac{16}{24}$.

Again, since $\frac{6}{12}=\frac{2}{4}$, $\frac{4}{8}=\frac{1}{2}$, etc., we deduce the following important Principle:

135. *If the two terms of any fraction be divided by the same number, the value of the fraction is not changed.*

Ex.—$\frac{10}{22}=\frac{5}{11}$; $\frac{12}{16}=\frac{3}{4}$.

Ex. 1.—Express $\frac{4}{5}$ as a fraction, having 15 for its denominator, or change $\frac{4}{5}$ to fifteenths.

To obtain 15 from 5, we must multiply 5 by 3, and since both terms of a fraction must be multiplied by the same number to have no effect on its value, we must multiply the numerator 4 by 3. This gives us the fraction $\frac{12}{15}$.

Thus $\frac{4}{5}=\frac{12}{15}$.

Ex. 2.—Change $\frac{10}{16}$ to eighths, that is, express $\frac{10}{16}$ as a fraction having its denominator 8.

To obtain 8 from 16 we divide 16 by 2, hence we must also divide 10 by 2, to give a new numerator.

Ans. $\frac{5}{8}$.

136. When the fraction $\frac{10}{16}$ is changed to $\frac{5}{8}$, the fraction $\frac{10}{16}$ is said to be brought to its lowest terms.

Instead of speaking of $\frac{10}{16}$ of a pound of sugar, we would express the same amount of sugar more easily by saying $\frac{5}{8}$ of a pound.

☞ All fractions should be expressed in their lowest terms.

137. A fraction is in its lowest terms when its numerator and denominator cannot both be divided exactly by any number greater than 1.

Reduce $\frac{8}{12}$ to its lowest terms. By dividing both terms by 2 we obtain $\frac{4}{6}$; again divide both terms of $\frac{4}{6}$ by 2 we have $\frac{2}{3}$, which we see is the required fraction.

In other words, $\frac{8}{12}$ of anything is the same as $\frac{2}{3}$ of it.

Instead of having two operations, we might have divided both terms of $\frac{8}{12}$ by 4, and thus find the lowest terms at once.

This number 4 we know is the G.C.D. of 8 and 12.

138. Hence to reduce a fraction to its lowest terms we have the following

RULE.

Divide both numerator and denominator of the fraction by their Greatest Common Divisor.

EXERCISE. 33.

(*a*) 1. Change $\frac{3}{4}$ to a fraction having its denominator 8; 12, 20; 40; 32; 60.

2. Express $\frac{5}{6}$ of a yard as twelfths; as sixteenths; as thirtieths.

3. How many twentieths of a dollar must I exchange for a half? for a quarter? for four-fifths? for seven-tenths?

4. Change

$\frac{5}{7}$ to thirty-fifths.
$\frac{5}{8}$ to fortieths.
$\frac{5}{6}$ to thirty-sixths.
$\frac{4}{5}$ to forty-fifths.

(*b*) Reduce the following fractions to their lowest terms:

1. $\frac{6}{10}$, $\frac{8}{20}$, $\frac{6}{8}$.
2. $\frac{14}{16}$, $\frac{15}{18}$, $\frac{5}{10}$.
3. $\frac{25}{35}$, $\frac{72}{88}$, $\frac{45}{63}$.
4. $\frac{84}{96}$, $\frac{33}{110}$, $\frac{65}{70}$.
5. $\frac{26}{32}$, $\frac{12}{72}$, $\frac{10}{1000}$.
6. $\frac{39}{57}$, $\frac{81}{900}$, $\frac{27}{407}$.
7. $\frac{28}{84}$, $\frac{70}{610}$, $\frac{27}{86}$.
8. $\frac{320}{480}$, $\frac{96}{232}$, $\frac{102}{144}$.
9. $\frac{63}{135}$, $\frac{29}{87}$, $\frac{91}{110}$.
10. $\frac{78}{198}$, $\frac{96}{144}$, $\frac{39}{44}$.

11. What fraction with lowest terms will express the same as $\frac{80}{120}$? $\frac{42}{84}$? $\frac{60}{90}$?

12. Shew that the fractions $\frac{36}{60}$, $\frac{21}{35}$, and $\frac{24}{40}$ are the same in value.

---

139. Hitherto we have spoken of such parts of the unit or whole as $\frac{3}{5}$, $\frac{5}{7}$, $\frac{5}{8}$, $\frac{7}{10}$, etc., that is, a part less than a whole in each case.

We now come to notice such fractions as $\frac{6}{5}$, $\frac{10}{7}$, $\frac{21}{8}$, $\frac{12}{10}$, etc.

Now $\frac{6}{5}$ means five-fifths and one-fifth, that is the whole and one-fifth besides, which is written $1\frac{1}{5}$ and read one and one-fifth. So $\frac{10}{7}$ means seven-sevenths and three-sevenths, or $1\frac{3}{7}$. $\frac{21}{8}$ = 16 eighths and 5 eighths = $2\frac{5}{8}$.

140. Such fractions as $\frac{3}{5}$, $\frac{5}{6}$, $\frac{6}{11}$, etc., in which the numerator is less than the denominator, are called **Proper Fractions.**

141. Such fractions as $\frac{5}{5}$, $\frac{10}{7}$, $\frac{21}{8}$, etc., in which the numerator is equal to or greater than the denominator are called **Improper Fractions.**

142. The expressions $2\frac{3}{8}$, $6\frac{1}{4}$, $8\frac{3}{5}$, etc., are called **Mixed Fractions,** being made up of a whole number and a fraction.

143. We see by Art. 139 that every improper fraction may be brought to a mixed fraction by means of the following

RULE.

Divide the numerator of the improper fraction by the denominator, and the quotient will be the whole number.

Place the remainder, if any, over the denominator, to form the other part of the mixed fraction.

Ex.—Reduce $\frac{2 3}{6}$ to a mixed fraction.

Dividing 23 by 6 we obtain 3 for the quotient and 5 for the remainder. Hence $3\frac{5}{6}$ will be the required mixed fraction.

In the mixed fraction $3\frac{5}{6}$ the 3 is the same as 18 sixths, which with the other 5 sixths make 23 sixths, $\frac{2 3}{6}$.

144. Hence, to reduce a mixed fraction to an improper fraction we have the following

RULE.

Multiply the whole number by the denominator, to the product add the numerator, and under the sum write the denominator.

Ex.—Reduce $7\frac{3}{5}$ to an improper fraction.

Multiplying 7 by 5, we have 35; adding 3 to this product gives 38; therefore the required fraction is $\frac{3 8}{5}$.

EXERCISE 54.

(*a*) Express the following improper fractions as mixed fractions, or whole numbers:

1. $\frac{1 9}{4}$.
2. $\frac{2 1}{3}$.
3. $\frac{2 8}{4}$.
4. $\frac{3 1}{5}$.
5. $\frac{1 9}{5}$.
6. $\frac{2 3}{4}$.
7. $\frac{3 1}{6}$.
8. $\frac{3 9}{8}$.
9. $\frac{4 5}{7}$.
10. $\frac{5 7}{8}$.
11. $\frac{7 3}{10}$.
12. $\frac{6 5}{5}$.
13. $\frac{9 3}{7}$.
14. $\frac{1 0 2}{6}$.
15. $\frac{1 2}{3}$.
16. $\frac{1 4}{6}$.
17. $\frac{1 5}{4}$.
18. $\frac{2 2}{3}$.
19. $\frac{1 7}{4}$.
20. $\frac{2 7}{4}$.
21. $\frac{3 5}{8}$.
22. $\frac{4 1}{1 0}$.
23. $\frac{5 4}{9}$.
24. $\frac{3 9}{4}$.
25. $\frac{4 9}{5}$.
26. $\frac{7 3}{6}$.
27. $\frac{8 4}{1 2}$.
28. $\frac{1 0 8}{6 7}$.
29. $\frac{6 4}{9}$.
30. $\frac{5 9}{8}$.
31. $\frac{7 5}{9}$.
32. $\frac{1 0 0}{2 0}$.
33. $\frac{2 4}{5}$.
34. $\frac{3 7}{6}$.
35. $\frac{2 9}{7}$.
36. $\frac{3 0}{5}$.
37. $\frac{3 2}{8}$.
38. $\frac{1 1 5}{8}$.
39. $\frac{1 2 9}{9}$.
40. $\frac{1 4 0}{8}$.
41. $\frac{2 0 0}{1 2}$.
42. $\frac{3 7 5}{1 5}$.
43. $\frac{1 9 8 6}{6 3}$.
44. $\frac{2 9 8 5}{4 7 6}$.

(*b*) Change the following mixed fractions to improper fractions:

1. $6\frac{2}{3}$.	12. $9\frac{5}{6}$.	23. $7\frac{5}{6}$.	34. $16\frac{3}{8}$.
2. $9\frac{1}{4}$.	13. $10\frac{3}{4}$.	24. $9\frac{5}{8}$.	35. $17\frac{5}{12}$.
3. $6\frac{3}{4}$.	14. $8\frac{3}{7}$.	25. $10\frac{3}{5}$.	36. $57\frac{3}{11}$.
4. $10\frac{1}{5}$.	15. $7\frac{4}{6}$.	26. $12\frac{5}{7}$.	37. $81\frac{5}{9}$.
5. $9\frac{1}{2}$.	16. $5\frac{3}{11}$.	27. $14\frac{3}{4}$.	38. $124\frac{1}{2}$.
6. $3\frac{2}{5}$.	17. $10\frac{6}{7}$.	28. $16\frac{2}{3}$.	39. $342\frac{2}{3}$.
7. $4\frac{1}{6}$.	18. $12\frac{1}{8}$.	29. $22\frac{4}{5}$.	40. $200\frac{5}{9}$.
8. $3\frac{7}{8}$.	19. $11\frac{1}{9}$.	30. $32\frac{3}{7}$.	41. $1256\frac{2}{7}$.
9. $5\frac{2}{3}$.	20. $9\frac{8}{9}$.	31. $45\frac{3}{5}$.	42. $4091\frac{7}{11}$.
10. $7\frac{3}{4}$.	21. $7\frac{6}{12}$.	32. $23\frac{3}{4}$.	
11. $8\frac{2}{5}$.	22. $8\frac{3}{7}$.	33. $5\frac{2}{7}$.	

(*c*) 1. How many whole days are there in $\frac{7}{8}$ of a day? In $\frac{4}{7}$ of a day?

2. How many whole ounces are there in $\frac{6}{9}$ of an ounce? In $\frac{111}{10}$ ounces? In $\frac{105}{12}$ of an ounce?

3. Take away as many dollars as you can from $\frac{6}{2}$ of a dollar, and what will be left? From $\frac{100}{11}$ of a dollar?

4. How many boys could receive a dollar apiece out of $\frac{485}{12}$ of a dollar? What part of a dollar would be left?

5. Change $5\frac{1}{4}$ to fourths.

6. Change 15 to fifths.

7. Reduce $13\frac{1}{6}$ to sixths.

8. Reduce $8\frac{3}{11}$ to elevenths.

9. Change $3\frac{1}{7}$ to sevenths. To fourteenths.

10. Change $5\frac{1}{9}$ to ninths. To eighteenths.

11. Change $6\frac{5}{12}$ to twelfths. To twenty-fourths.

12. Change $8\frac{3}{4}$ to fourteenths. To forty-seconds.

13. Reduce $9\frac{5}{20}$ to twentieths. To sixtieths.

14. How many eighths of a pound are there in $25\frac{3}{8}$ pounds? In $30\frac{1}{2}$ pounds? In $100\frac{1}{4}$ pounds?

15. How many more quarters of a yard are there in $85\frac{3}{4}$ yards than in $32\frac{1}{4}$ yards?

145. If we take the two fractions $\frac{7}{12}$ and $\frac{5}{8}$, and wish to know the greater of the two, it might not be known from a glance at the fractions. We will therefore find a method of comparing fractions, that is, finding out the order in which they stand according to their value.

We know that $\frac{7}{12} = \frac{14}{24}$.
Also, $\frac{5}{8} = \frac{15}{24}$.

Now we have the two fractions $\frac{14}{24}$ and $\frac{15}{24}$, in each of which the size of the parts of the unit is the same, that is, twenty-fourths; but in the first there are 14 and in the second 15 of these twenty-fourths taken. Hence we see at a glance that $\frac{15}{24}$ is greater than $\frac{14}{24}$ by $\frac{1}{24}$, that is, $\frac{5}{8}$ is greater than $\frac{7}{12}$ by $\frac{1}{24}$.

Thus, a boy who received $\frac{5}{8}$ of a lot of apples would have more than a boy who obtained $\frac{7}{12}$ of the lot.

146. In order to compare the two fractions, they were brought to other fractions, having the same denominator, 24. This is the method we were seeking, and applies to any number of fractions.

☞ This same denominator, it will be easily noticed, is the L. C. D. of the given denominators.

147. We thus have the following

RULE FOR COMPARING FRACTIONS.

Find the L. C. D. of all the given denominators.

Bring each fraction to another having the L. C. D. for a new denominator.

The value of the fractions will then depend entirely on the value of the new numerators.

☞ Mixed fractions must be first brought to improper fractions.

Ex.—Find the greatest and least of the following: $\frac{2}{3}$ of a pound, $\frac{5}{12}$ of a pound, $\frac{7}{9}$ of a pound.

The L. C. D. of the denominators 3, 12 and 9 is 36.

$\frac{2}{3}$ of a pound $= \frac{24}{36}$ of a pound.
$\frac{5}{12}$ of a pound $= \frac{15}{36}$ of a pound.
$\frac{7}{9}$ of a pound $= \frac{28}{36}$ of a pound.

Hence we have 24 parts, 15 parts, and 28 parts of a pound. Therefore $\frac{28}{36}$ or $\frac{7}{9}$ of a pound is the greatest, $\frac{24}{36}$ or $\frac{2}{3}$ of a pound is the next, and $\frac{15}{36}$ or $\frac{5}{12}$ of a pound is the least.

EXERCISE 35.

(*a*) Arrange the following fractions in their order of value:

1.	$\frac{1}{2}$.	$\frac{2}{3}$.	$\frac{5}{6}$.	$\frac{7}{12}$.	5.	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{2}{9}$	$\frac{23}{24}$.
2.	$\frac{4}{5}$.	$\frac{3}{10}$.	$\frac{7}{20}$.	$\frac{3}{2}$.	6.	$\frac{2}{3}$.	$\frac{7}{8}$.	$\frac{11}{12}$.	$\frac{19}{20}$.
3.	$\frac{3}{6}$.	$\frac{11}{12}$.	$\frac{1}{18}$.	$\frac{4}{9}$.	7.	$\frac{3}{15}$.	$2\frac{1}{4}$.	$\frac{9}{10}$.	$\frac{13}{25}$.
4.	$\frac{4}{9}$.	$\frac{3}{5}$.	$\frac{8}{15}$.	$\frac{2}{3}$.	8.	$\frac{1}{2}$.	$3\frac{1}{4}$.	6.	$\frac{5}{8}$.

(*b*) 1. A. owns $\frac{1}{2}$ a ton, B. $\frac{2}{3}$ of a ton, C. $\frac{7}{8}$ of a ton, and D. $\frac{5}{8}$ of a ton: who has the most, and who the least?

2. A. does $\frac{5}{8}$ of a day's work, B. $\frac{2}{5}$, C. $\frac{3}{4}$, and D. $\frac{4}{15}$: which should draw the most pay? Which the least?

3. John has $\$3\frac{5}{8}$, James $\$3\frac{4}{7}$, Henry $\$5\frac{3}{8}$, Charles $\$9\frac{4}{14}$: arrange their names in the order of their wealth.

4. A man rides 19 quarters of a mile, drives $3\frac{11}{12}$ miles, walks $\frac{6}{84}$ of a mile, and sails $\frac{3}{7}$ of a mile: which was the longest, and which the shortest?

5. A pole is $\frac{31}{7}$ feet high, another $8\frac{6}{14}$, another $\frac{6}{7}$, and another $9\frac{2}{5}$: arrange them in order of height.

6. Which is the greatest and which the least of the following amounts: $\$9\frac{4}{5}$, $\$3\frac{14}{28}$, $\$4\frac{8}{40}$, $\$\frac{3}{7}$? Arrange them in order of value.

ADDITION OF FRACTIONS.

148. We now proceed to add fractions when the fractions have denominators that are alike.

In this case, since the parts of the unit or whole are the same in each fraction, the sum of the numerators will show the number of parts there are altogether.

If we add $\frac{3}{7}$ of a pound to $\frac{2}{7}$ of a pound, or 2 and 3 of the same size parts (sevenths), we must obtain 5 of these sevenths, that is $\frac{5}{7}$ of a pound.

149. Hence we have the following

RULE.

Add together the several numerators, and under the sum place the given denominator.

Ex.—Add together $\frac{1}{12}$, $\frac{5}{12}$, and $\frac{7}{12}$.

The sum of the numerators is 13, so the required fraction must be $\frac{13}{12}$.

To add fractions when the fractions have denominators that are unlike.

In this case, since the size of the parts of the unit or whole is not the same in each fraction, the fractions must be brought to other fractions having the same denominator. Then we proceed as in the former case.

150. This gives the following

RULE.

Reduce all the fractions to others having the same denominator.

Add the new numerators, and under their sum place the new denominator.

Ex. 1.—Find the sum of $\frac{3}{10}$ and $\frac{5}{12}$.

$$\frac{3}{10} + \frac{5}{12} = \frac{18 + 25}{60} = \frac{43}{60}$$

Here $\frac{3}{10} = \frac{18}{60}$, $\frac{5}{12} = \frac{25}{60}$, and the sum is $\frac{43}{60}$.

☞ If there be any mixed fractions, the whole numbers are added separately, and then the sum of the fractions is added to that sum.

Ex. 2.—Add $\$5\frac{1}{10}$, $\$7\frac{1}{8}$, $\$10\frac{1}{4}$.

$$5 + 7 + 10 = 22.$$

$$\frac{1}{10} + \frac{1}{8} + \frac{1}{4} = \frac{4+5+10}{40} = \frac{19}{40}$$

$$\$22 + \$\frac{19}{40} = \$22\frac{19}{40}.$$

SUBTRACTION OF FRACTIONS.

151. In subtracting one fraction from another, the same remarks apply as in the addition of fractions, except that instead of adding the numerators we subtract them.

Ex. 1.—$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.

$$\frac{5}{12} - \frac{3}{10} = \frac{25 - 18}{60} = \frac{7}{60}$$

Ex. 2—From $23\frac{5}{12}$ subtract $17\frac{3}{4}$.

$23\frac{5}{12}$
$17\frac{3}{4}$
$5\frac{8}{12}$

In this case we cannot take $\frac{3}{4}$ from $\frac{5}{12}$, that is $\frac{9}{12}$ from $\frac{5}{12}$; so, as in ordinary subtraction, we must use one of the next higher order, that is, 1 unit, or $\frac{12}{12}$. Then $\frac{12}{12}+\frac{5}{12}=\frac{17}{12}$, and $\frac{9}{12}$ from $\frac{17}{12}$ leaves $\frac{8}{12}$. Again, since we used one of the 23 units, there are only 22 left, and 17 from 22 leaves 5, giving the final result $5\frac{8}{12}$, or $5\frac{2}{3}$.

The work might be written in the following form:

$23\frac{5}{12} = 22 + 1\frac{5}{12} = 22 + \frac{17}{12}$
$17\frac{3}{4} = 17 + \frac{3}{4} = 17 + \frac{9}{12}$
Difference $= 5 + \frac{8}{12} = 5\frac{2}{3}$.

☞ In giving a result or answer, improper fractions should (unless otherwise stated) be brought to mixed fractions, and all fractions reduced to their lowest terms.

EXERCISE 36.

☞ These questions should be solved mentally.

1. Add together and find the difference between: $\frac{1}{2}$ and $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{5}{4}$; $\frac{2}{7}$ and $\frac{3}{7}$; $\frac{5}{6}$ and $\frac{1}{6}$; $\frac{1}{3}$ and $\frac{1}{4}$; \$$2\frac{1}{2}$ and \$$3\frac{1}{2}$; $\frac{3}{8}$ and $\frac{1}{2}$; $\frac{5}{6}$ of a foot and $\frac{7}{8}$ of a foot; $\frac{3}{8}$ of a farm and $\frac{1}{4}$ of a farm; $\frac{5}{12}$ and $\frac{1}{6}$; $\frac{9}{15}$ and $\frac{1}{5}$; $\frac{1}{6}$ of a share and $\frac{1}{6}$ of a share; $\frac{7}{12}$ and $\frac{1}{3}$.

2. Add together $\frac{2}{3}$ and $1\frac{1}{12}$; $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$; $\frac{3}{4}$ of a yard, $\frac{1}{8}$ of a yard, $\frac{1}{2}$ of a yard; $\frac{1}{6}$, $\frac{1}{10}$, $\frac{3}{20}$.

3. John had $\frac{5}{8}$ of an apple, and gave away $\frac{1}{4}$ of the apple: what had he left?

4. Bought goods for $\frac{3}{4}$ of a dollar, and sold them for $1\frac{1}{2}$ dollars: what did I gain?

5. A boy paid $\frac{1}{8}$ of a dollar for a book, $\frac{1}{2}$ a dollar for a bag, $\frac{1}{4}$ of a dollar for paints: how much did he spend?

6. Find the difference between, and also the sum of $\frac{4}{5}$ and $\frac{2}{5}$; $\frac{1}{6}$ and $\frac{5}{6}$; $\frac{3}{7}$ and $\frac{2}{3}$; $\frac{3}{4}$ and $\frac{2}{5}$; $3\frac{1}{2}$ and $2\frac{1}{4}$; $5\frac{3}{4}$ and $4\frac{1}{8}$; $3\frac{3}{7}$ and $1\frac{5}{7}$; $2\frac{1}{6}$ inches and $1\frac{1}{3}$ inches; $5\frac{1}{2}$ and $3\frac{3}{8}$; $7\frac{1}{10}$ and $3\frac{2}{5}$; $5\frac{1}{2}$ yards and $2\frac{3}{4}$ yards; $6\frac{1}{2}$ ounces and $4\frac{5}{8}$ ounces; $17\frac{1}{8}$ cents and $12\frac{7}{8}$ cents; 20 $\frac{1}{11}$ oranges and $15\frac{10}{11}$ oranges; $7\frac{3}{10}$ dollars and $5\frac{5}{8}$ dollars.

7. What must I add to $\$1\frac{3}{4}$ to make $\$3\frac{1}{2}$?

8. What must I take from $\$7\frac{1}{2}$ to make $\$3\frac{3}{4}$?

9. From what must $3\frac{1}{2}$ be taken to leave $2\frac{1}{4}$?

10. What would be left if $3\frac{5}{8}$ yards were taken from $6\frac{1}{4}$ yards?

11. From what must we cut off $5\frac{3}{5}$ yards to leave $3\frac{2}{5}$ yards?

12. To what must $5\frac{1}{8}$ cents be added to produce $10\frac{1}{2}$ cents?

13. What part of a farm must be sold to leave $\frac{1}{10}$ of it?

14. What must be taken from a square rod of land ($30\frac{1}{4}$ yards) to leave $15\frac{1}{2}$ yards?

---

152. The pupil will have noticed that in adding such fractions as $\frac{1}{7}$ and $\frac{1}{8}$ the result, $\frac{15}{56}$, is just the sum of the denominators placed over their product. The difference of the fractions would be the difference of their denominators placed over their product. For example $\frac{1}{11}-\frac{1}{12}=\frac{1}{132}$.

This may be stated as follows:

The sum or difference of two fractions, having 1 *for a numerator, is the sum or difference of their denominators, placed over their product.*

EXERCISE. 37.

1. $4\frac{1}{5}+3\frac{1}{7}$.
2. $19\frac{1}{4}+13\frac{1}{18}$.
3. $8\frac{1}{2}+6\frac{1}{3}$.
4. $25\frac{3}{4}+14\frac{1}{2}$.
5. $20\frac{3}{8}-16\frac{1}{2}$.
6. $18\frac{1}{4}-12\frac{1}{7}$.
7. $22\frac{7}{12}+18\frac{1}{5}$.
8. $21\frac{1}{2}+15\frac{1}{7}$.
9. $33\frac{1}{3}+24\frac{1}{4}$.
10. $48\frac{7}{8}-26\frac{7}{16}$.
11. $75\frac{6}{15}-36\frac{6}{21}$.
12. $1\frac{1}{10}+6\frac{2}{5}+18\frac{9}{20}+2\frac{7}{30}$.
13. $46\frac{5}{8}-15\frac{1}{6}$.
14. $13\frac{1}{7}+99\frac{3}{8}+512\frac{5}{14}$.
15. $3\frac{4}{15}-1\frac{2}{9}$.
16. $\frac{7}{25}-\frac{7}{35}$.
17. $4\frac{2}{5}+\frac{7}{18}\frac{5}{5}+3\frac{1}{9}$.
18. $\frac{2}{80}\frac{7}{}-\frac{5}{24}$.

19. A boy had 4 acres; his brother gave him $\frac{5}{6}$ of another, his sister $\frac{3}{4}$ of another, and his father $\frac{3}{8}$ of another: how many had he then?

20. A merchant had a piece of calico containing $34\frac{3}{4}$ yards, and sold off it $21\frac{3}{5}$ yards: how much was left?

21. A man paid \$11 for a coat, $\$\frac{8}{3}$ for a knife, $\$\frac{5}{8}$ for a brush, and $\$\frac{5}{9}$ for a comb: how much did they all cost?

22. A man bought $17\frac{5}{6}$ pounds of butter, from which he sold $12\frac{1}{2}$ pounds: how much remained?

23. From a cask containing $42\frac{1}{4}$ gallons of molasses, a grocer drew off $17\frac{1}{2}$ gallons: how many gallons remained?

24. A man bought a horse for $\$73\frac{3}{9}$, a carriage for $\$97\frac{5}{9}$, a set of harness for $\$37\frac{5}{12}$: what was the cost of the whole?

25. A man gave away at different times $\$\frac{5}{7}$, $\$\frac{3}{9}$, $\$\frac{8}{21}$, and $\$1\frac{2}{4}$: how much did he give away in all?

26. A man started on a journey of $45\frac{2}{3}$ miles, and travelled $28\frac{7}{8}$ miles: how far had he still to travel?

27. A merchant bought $272\frac{2}{3}$ yards of muslin from one man, $117\frac{3}{4}$ yards from another, and $321\frac{5}{6}$ yards from a third: how many yards did he buy in all?

28. James has 3 fishing lines; the first measures $12\frac{1}{2}$ feet, the second $14\frac{1}{4}$ feet, and the third $15\frac{5}{8}$ feet: how many feet in the 3 lines?

29. A merchant bought 3 pieces of calico, the first containing $25\frac{3}{4}$ yards, the second $22\frac{1}{2}$ yards, and the third $34\frac{1}{4}$ yards: how many yards are there in the 3 pieces?

30. If I have $\$437\frac{3}{9}$, and pay out $\$341\frac{4}{15}$, how much have I left?

31. A lady having \$100, paid $\$8\frac{1}{4}$ for a pocket handkerchief, $\$15\frac{1}{2}$ for a dress hat, $\$46\frac{5}{8}$ for a cloak: how much had she left?

32. A clerk earned $\$50\frac{1}{2}$ per month. He paid $\$20\frac{3}{4}$ for board, $\$5\frac{3}{4}$ for washing, and $\$4\frac{3}{5}$ for other expenses: how much did he save per month?

33. What number added to $147\frac{4}{7}$ will make $216\frac{8}{9}$?

34. What number added to $307\frac{1}{7}+210\frac{3}{4}$ will make $700\frac{5}{8}$?

35. What number must be added to the difference of $186\frac{5}{9}$ and $214\frac{3}{7}$ to make $1042\frac{19}{21}$?

36. What fraction added to the sum of $\frac{1}{8}$, $\frac{5}{12}$, and $\frac{4}{18}$, will make $\frac{133}{144}$?

37. Bought a quantity of barrel staves for $\$160\frac{3}{8}$, and lumber for $\$1136\frac{2}{3}$: I sold the staves for $\$205\frac{1}{2}$, and the lumber for $\$1240\frac{9}{16}$: what was my whole gain?

38. A man bought a ton of hay for $\$15\frac{3}{8}$, a barrel of flour for $\$9\frac{5}{12}$, and a barrel of apples for $\$3\frac{7}{16}$: what change should be given to him for 3 ten-dollar bills?

39. What must be taken from $35\frac{1}{2}$ to leave $22\frac{2}{5}$?

40. From what must $24\frac{4}{7}$ be taken to leave $63\frac{2}{3}\frac{2}{5}$?

41. Bought of Davison, Scott & Co. four cheeses weighing $46\frac{5}{8}$, $48\frac{2}{3}$, $49\frac{7}{16}$, and $57\frac{1}{4}$ pounds respectively: what was their whole weight?

42. A pole has three-eighths of its length painted red, two-fifths of it white, and the rest of it blue: what part of its length is blue?

MULTIPLICATION OF FRACTIONS.

153. To multiply a fraction by a whole number when the denominator does not contain the whole number exactly.

Multiply $\frac{3}{5}$ by 7. Here we have simply to find what 3 parts become when repeated 7 times, the parts being fifths. The result will be 21 parts or fifths, which is written $\frac{21}{5}$.

Therefore $\frac{3}{5} \times 7 = \frac{21}{5}$.

When the denominator does contain the whole number exactly.

Multiply $\frac{3}{10}$ by 5. Here, instead of repeating the number of parts, we will increase their size 5 times, that is, make the tenths become halves, since five-tenths make one-half, and the result will be $\frac{3}{2}$, taking the same number of parts we had before.

Therefore $\frac{3}{10} \times 5 = \frac{3}{2}$.

154. From the preceding examples we deduce the following

RULE.

To multiply a fraction by a whole number, divide the denominator by the whole number, if possible, and keep the same numerator; or multiply the numerator by the whole number, and keep the same denominator.

Ex. 1.—$\frac{5}{12} \times 3 = \frac{5}{4}$.

Ex. 2.— $\frac{2}{5} \times 3 = \frac{6}{5}$.

155. To multiply a mixed fraction by a whole number.

What will 12 pounds of sugar cost at $11\frac{3}{5}$ cents a pound? Or, multiply $11\frac{3}{5}$ cents by 12.

cents
$11\frac{3}{5}$
12
$139\frac{1}{5}$

The $11\frac{3}{5}$ cents may be broken into two parts—11 cents and $\frac{3}{5}$ cents. We first multiply the $\frac{3}{5}$ by 12. This gives $\frac{36}{5}$, which is equal to $7\frac{1}{5}$ cents. We write down the $\frac{1}{5}$ and carry the 7 cents to the next order to the left as usual. Then $11 \times 12 = 132$; 132 added to the 7 cents makes 139 cents. The result will thus be $\$1.39\frac{1}{5}$.

156. From this we have the following

RULE.

To multiply a mixed fraction by a whole number, multiply the fractional part by the multiplier, and reduce this, if possible, to a mixed fraction.

Then multiply the other part of the mixed fraction by the multiplier, and add to the product the part carried, if any.

EXERCISE 38.

☞ These questions should be solved mentally.

1. Multiply $\frac{5}{12}$ of a dollar by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
2. Multiply $\frac{5}{16}$ of a pound by 2, 6, 8, 12, 10.
3. If a basket holds $\frac{3}{10}$ of a bushel of apples, how much can be put into 3 baskets? 5 baskets? 7 baskets? 10 baskets?
4. If each man receive $\$2\frac{3}{8}$, how much will 3 men receive? 6 men? 8 men?
5. At $2\frac{3}{4}$ cents a yard, find the cost of 3 yards; 6 yards; 4 yards.
6. What will $\frac{5}{8}$ of a ton of hay cost at $\$24$ a ton?
7. How many units in 6 times $\frac{1}{8}$? In 8 times $\frac{5}{10}$? In 11 times $\frac{4}{7}$?
8. At $6\frac{1}{4}$ cents each, what will 12 pencils cost?
9. Find the cost of 7 dozen pens at $10\frac{1}{4}$ cents a dozen?
10. A man earns $\$5\frac{3}{4}$ a week: what will he earn in a month (4 weeks)?
11. Find the cost of 10 citrons at $15\frac{1}{2}$ cents each.
12. What will 8 cords of wood cost at $\$5\frac{1}{16}$ a cord?
13. How far can I travel in 10 hours at the rate of $6\frac{3}{5}$ mile an hour?
14. A horse eats $2\frac{1}{2}$ bushels of oats in one week: how much will he eat in two months?

DIVISION OF FRACTIONS.

157. To divide a fraction by a whole number when the numerator can be exactly divided by the whole number.

If 4 men earn $\frac{8}{9}$ of a dollar, how much is that each? This may be written: If 4 men earn 8 parts, how much will 1 man earn? The fourth part of 8 parts, that is, 2 parts, or ninths.

Therefore $\$\frac{8}{9} \div 4 = \$\frac{2}{9}$.

When the numerator does not exactly contain the whole number.

Divide $\frac{3}{4}$ of an apple into 5 equal parts, that is, divide $\frac{3}{4}$ by 5. Here, since we cannot divide the 3 parts exactly by 5, we must decrease the size of the parts, making each part five times smaller, that is, twentieths.

Therefore $\frac{3}{4} \div 5 = \frac{3}{20}$.

158. This gives us the following

RULE.

To divide a fraction by a whole number, divide the numerator, if possible, by the whole number, keeping the same denominator; or multiply the denominator by the whole number, keeping the same numerator.

Ex. 1.—$\frac{6}{7} \div 2 = \frac{3}{7}$.

Ex. 2.—$\frac{3}{5} \div 2 = \frac{3}{10}$.

159. To divide a mixed fraction by a whole number.

$5 \overline{)\ 13\frac{3}{4}}$ $\quad 2\frac{3}{4}$

Divide $\$13\frac{3}{4}$ equally among 5 men. 5 is contained in 13, 2 times and 3 over. This 3 with the $\frac{3}{4}$ makes $\frac{15}{4}$, and by the preceding principle the fifth part of $\frac{15}{4}$ is $\frac{3}{4}$. Hence the result must be $\$2\frac{3}{4}$.

$5 \overline{)\ 12\frac{3}{5}}$ $\quad 2\frac{13}{25}$

Ex. 2.—Divide $12\frac{3}{5}$ by 5. $12 \div 5 = 2$, with 2 remaining. $2\frac{3}{5} = \frac{13}{5}$. $\frac{13}{5} \div 5 = \frac{13}{25}$. Therefore, $12\frac{3}{5} \div 5 = 2\frac{13}{25}$.

160. From this we deduce the following

RULE.

To divide a mixed fraction by a whole number, divide the whole number of the mixed fraction by the divisor, if possible, reduce the remaining part, if any, to an improper fraction, then divide as usual.

Ex.—Divide $15\frac{3}{4}$ by 5.

$15 \div 5 = 3$.

$\frac{3}{4} \div 5 = \frac{3}{20}$. Therefore $15\frac{3}{4} \div 5 = 3\frac{3}{20}$. *Ans.*

EXERCISE 39.

☞ These questions should be solved mentally.

1. Divide $\frac{5}{9}$ of a dollar by 2, 3, 4, 5, 6, 7, 8.
2. Find the third part of $\frac{10}{11}$ of a pound; the fifth part; the eighth part.
3. What does each man get if $\frac{9}{10}$ of an acre of land be divided among 3 men? Among 4 men? Among 6 men? Among 9 men?
4. Four men can earn $\$12\frac{1}{2}$ a day: what will one man earn? Three men? Six men?
5. At the rate of $\$\frac{7}{8}$ for 3 bushels, what will one bushel of oats cost? Seven bushels?
6. A man had 10 gallons of cream, and sold $\frac{1}{2}$ of it for $\$\frac{15}{16}$: how much a gallon did he get? How much a quart?
7. I sold 3 barrels of meal for $\$20\frac{1}{2}$: what did I get a barrel?
8. A man pays $\$12\frac{1}{2}$ for 5 sheep: how much was it apiece? What would be the cost of 2? Of 3? Of 4?
9. How often is 6 contained in $13\frac{1}{2}$? In $15\frac{1}{3}$? In $12\frac{6}{7}$? In $11\frac{1}{7}$?
10. There are $5\frac{1}{2}$ yards in a rod of fencing: how many yards in a quarter of a rod? In six rods?
11. I bought 5 barrels of flour for $\$15\frac{3}{8}$, in selling I gain $\$5\frac{1}{4}$: what did I sell it at per barrel? If I lost $\$5\frac{1}{4}$, how much did I sell each barrel for?

12. What are apples worth each at $15\frac{3}{7}$ cents a dozen? At $20\frac{4}{10}$ cents a dozen?

13. A gallon contains 32 gills: how many gills are there in $\frac{7}{8}$ of a gallon? In $\frac{3}{4}$ of a gallon? In $\frac{5}{16}$ of a gallon?

14. A rod being $5\frac{1}{2}$ yards in length, how many yards in 3 rods? In 10 rods?

---

161. To multiply a fraction by a fraction.

Ex.—Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

$\frac{1}{4}$ of $\frac{5}{7}$, or a quarter of $\frac{5}{7}$, is $\frac{5}{28}$, and $\frac{3}{4}$ of $\frac{5}{7}$, or three-quarters of $\frac{5}{7}$, will be 3 times $\frac{5}{28}$, or $\frac{15}{28}$, which equals $\frac{3\times5}{4\times7}$ that is, equal to the product of the numerators placed over the product of the denominators of the given fractions.

The expression $\frac{3}{4}$ of $\frac{5}{7}$ is called a **Compound Fraction.**

162. A **Compound Fraction** may thus be called a fraction of a fraction.

163. From the above we have the following

RULE.

To multiply fractions together, or to simplify compound fractions, first reduce each mixed fraction, if any, to an improper fraction. Then multiply the numerators together for a new numerator, and the denominators together for a new denominator, always reducing the resulting fraction to its lowest terms.

Ex. 1.—Find the product of $\frac{5}{12}$ and $\frac{9}{20}$.

$$\frac{5}{12}\times\frac{9}{20}=\frac{5\times9}{12\times20}=\frac{45}{240}=\frac{3}{16}$$

The product of the numerators is 45, and the product of the denominators is 240, and the fraction $\frac{45}{240}$, when reduced to its lowest terms, is $\frac{3}{16}$, which is therefore the required result.

164. In the above example, instead of finding the fraction $\frac{45}{240}$ and then reducing it to its lowest terms, we may reduce the fraction $\frac{5 \times 9}{12 \times 20}$ to its lowest terms at once by dividing both terms of the fraction by the same number, thus:

$$\frac{\overset{1}{\cancel{5}} \times \overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}} \times \underset{4}{\cancel{20}}}$$

5 is contained in itself once, and in 20 four times. Again 3 will divide 9 three times, and 12 four times. Now multiply as before and we obtain $\frac{1 \times 3}{4 \times 4} = \frac{3}{16}$

165. This method is always used when possible, and the operation is known by the name of **Cancelling**.

Ex. 2.—Simplify $\frac{5}{6}$ of $\frac{7}{8} \times \frac{4}{9}$ of $\frac{10}{21} \times 2\frac{2}{5}$.

This expression may be written:

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{2}{\cancel{9}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{21}}} \times \frac{\overset{1}{\cancel{12}}}{\underset{1}{\cancel{5}}} = \frac{10}{27}$$

Ex. 3.—If a pound of sugar costs $6\frac{2}{7}$ cents, what must I pay for $3\frac{2}{11}$ pounds?

If one pound cost $6\frac{2}{7}$ cents, the cost of $3\frac{2}{11}$ will be $3\frac{2}{11}$ times $6\frac{2}{7}$ cents.

$$6\tfrac{2}{7} \times 3\tfrac{2}{11} = \frac{\overset{4}{\cancel{44}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{5}{\cancel{55}}}{\underset{1}{\cancel{11}}} = 20 \text{ cents.}$$

EXERCISE 40.

1. Find the product of $\frac{4}{5}$ and $\frac{7}{8}$.
2. Multiply $\frac{6}{11}$ by $\frac{9}{22}$.
3. Multiply together $\frac{29}{36}$, $\frac{5}{6}$, $\frac{27}{58}$.
4. Find the value of $1\frac{1}{19}$ of $1\frac{1}{18}$ of $1\frac{1}{17}$.
5. Simplify $10\frac{1}{2} \times 2\frac{2}{3}$.
6. Simplify $\frac{9}{14}$ of $6\frac{1}{2}$ of $1\frac{1}{3} \times \frac{15}{16} \times \frac{7}{13}$ of $1\frac{1}{5}$.
7. Multiply $\frac{9}{14}$ of $2\frac{1}{2}$ of $3\frac{1}{9}$ by $6\frac{1}{2}$ of $7\frac{1}{3}$.

8. Simplify $\frac{7}{18}$ of $\frac{3}{14}$ of $7\frac{1}{8} \times 3\frac{1}{8}$ of $\frac{20}{187}$ of $\frac{34}{5}$.

9. What will $3\frac{3}{5}$ pounds of rice cost at $4\frac{1}{2}$ cents a pound?

10. Find the value of $4\frac{4}{5}$ pounds of butter at $27\frac{1}{4}$ cents per pound.

11. Find the cost of $26\frac{1}{4}$ ounces of candy at $5\frac{1}{5}$ cents an ounce.

12. What must I pay for $4\frac{1}{2}$ bushels of clover seed at $\$7\frac{5}{6}$ a bushel?

13. How far can a man, walking $2\frac{2}{9}$ miles an hour, go in $17\frac{1}{2}$ hours?

14. What number divided by $4\frac{2}{3}$ will give $8\frac{5}{8}$ as a result?

15. What does A. earn in $9\frac{3}{5}$ weeks at the rate of $\$23\frac{1}{3}$ per week?

16. What is $\frac{3}{5}$ of $\$17\frac{1}{4}$?

17. With a machine, a man cuts $8\frac{1}{4}$ cord of maple a day: what quantity can he cut in $4\frac{2}{3}$ days at the same rate?

18. What will $33\frac{1}{3}$ pounds of tea cost at $93\frac{3}{4}$ cents a pound?

19. What will $212\frac{2}{3}$ pounds of meat cost at $7\frac{1}{2}$ cents a pound?

20. How many cords of wood will 45 men chop in $15\frac{1}{2}$ days, if each chop $2\frac{4}{5}$ cords a day?

21. Find the amount of the following account: $7\frac{1}{2}$ lbs. of rice @ $5\frac{3}{4}$cts. a pound; $9\frac{7}{8}$ quarts of beans @ $7\frac{2}{5}$cts. a quart; $12\frac{1}{2}$ yards of ribbon @ $6\frac{7}{5}$cts. a yard; $9\frac{3}{4}$ yards of linen @ $7\frac{2}{3}$cts. a yard; $12\frac{1}{4}$ yards of lace @ $62\frac{1}{2}$cts. a yard.

22. A trunk cost $\frac{1}{4}$ of $\$16\frac{3}{4}$, and a valise $\frac{8}{9}$ as much as the trunk; what did I pay for the valise?

23. Multiply 18 times $\frac{7}{9}$ of $5\frac{1}{4}$ by $\frac{1}{6}$ of 4 times $\frac{5}{6}$ of 3.

24. Find the cost of $\frac{1}{3}$ of 5 yards of lace at $\frac{3}{4}$ of $\$18\frac{1}{2}$ a yard.

25. What must be paid for $\frac{5}{6}$ of $156\frac{2}{3}$ acres of land at $\frac{4}{5}$ of $\$54\frac{9}{10}$ an acre?

26. The yacht "Oriole" can make $10\frac{2}{7}$ knots an hour: how many knots would she sail in $4\frac{1}{12}$ hours?

166. To divide a fraction by a fraction.

Divide $\frac{5}{7}$ by $\frac{3}{8}$.

In $\frac{5}{7}$, 1 unit is contained **five-sevenths** of a time. Then **one-eighth** of 1 unit must be contained **eight** times as often, that is, $\frac{40}{7}$ times. Now, **three-eighths** of 1 unit will be contained **one-third** as many times as **one-eighth**, that is, $\frac{40}{21}$ of a time.

And $\frac{40}{21} = \frac{5 \times 8}{7 \times 3}$

This result will be seen to be the same as that obtained by multiplying the first fraction by the latter inverted, that is, with the places of its two terms changed.

167. This gives us the following

RULE.

To divide a fraction by a fraction, reduce mixed and compound fractions, if any, to simple fractions. Then invert the divisor and multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Ex. 1.—Divide $\frac{7}{9}$ of $\frac{3}{4}$ of $\frac{3}{7}$ by $\frac{1}{4}$ of $\frac{3}{4}$.

$$\frac{\cancel{7}^{1}}{\cancel{9}_{\cancel{3}_{1}}} \text{ of } \frac{\cancel{3}^{1}}{\cancel{4}} \text{ of } \frac{\cancel{3}^{1}}{\cancel{7}_{1}} = \frac{1}{4}$$

$$\frac{1}{7} \text{ of } \frac{3}{4} = \frac{3}{28}$$

$$\frac{1}{4} \div \frac{3}{28} = \frac{1}{\cancel{4}_{1}} \times \frac{\overset{7}{\cancel{28}}}{3} = \frac{7}{3} = 2\tfrac{1}{3}$$

The dividend becomes $\frac{1}{4}$, the divisor $\frac{3}{28}$, which being inverted is $\frac{28}{3}$. Then $\frac{1}{4}$ multiplied by $\frac{28}{3}$ gives $\frac{7}{3}$, or $2\frac{1}{3}$, the required result.

Ex. 2.—How many pieces of cloth $\frac{2}{15}$ of a yard long can be cut from $\frac{1}{2}$ of $\frac{9}{10}$ of a yard?

There must be as many pieces of cloth as the number of times that $\frac{2}{15}$ is contained in $\frac{1}{2}$ of $\frac{9}{10}$.

$$\frac{1}{2} \text{ of } \frac{9}{10} \div \frac{2}{15} = \frac{9}{\cancel{20}_{4}} \times \frac{\overset{3}{\cancel{15}}}{2} = \frac{27}{8} = 3\tfrac{3}{8} \text{ pieces.}$$

EXERCISE 41.

1. Divide $\frac{24}{65}$ by $\frac{8}{13}$.
2. Divide $\frac{18}{49}$ by $\frac{6}{7}$.
3. Divide $\frac{36}{91}$ by $\frac{9}{13}$.
4. Find the quotient of $\frac{26}{76} \div \frac{13}{25}$.
5. How often is $2\frac{1}{2}$ contained in $6\frac{1}{4}$?
6. How many times $2\frac{1}{3}$ is $18\frac{2}{3}$?
7. What number must be multiplied by $1\frac{1}{4}$ to give $7\frac{1}{2}$?
8. The dividend is $12\frac{1}{4}$; quotient, $3\frac{1}{2}$; find the divisor.
9. Divide $\frac{3}{7}$ of $\frac{5}{8}$ of 16 by $\frac{3}{5}$ of $\frac{6}{7}$ of $5\frac{1}{3}$.
10. How many times $\frac{7}{9}$ of $\frac{3}{4}$ of $\frac{5}{7}$ is $\frac{1}{7}$ of $\frac{3}{4}$?
11. Find the quotient of $\frac{3}{14}$ of $3\frac{2}{3} \times 6$ divided by $1\frac{2}{7}$ of 6 times $\frac{7}{9}$.
12. How many yards of silk can be bought for $\$317\frac{2}{3}$ at $\$3\frac{2}{5}$ a yard?
13. A man earns $\frac{3}{8}$ of a dollar an hour: in how many hours will he earn $\$17\frac{1}{4}$?
14. How many times $\frac{11}{23}$ of a dollar is $\$176$?
15. Divide $\frac{45}{28}$ of $\frac{21}{15}$ of $\frac{9}{12}$ of $\frac{10}{13}$ by $\frac{3}{28}$ of $\frac{30}{36}$ of $\frac{3}{4}$ of 5.
16. How much tea at $\$1\frac{7}{8}$ a pound can I buy with $\$57\frac{3}{4}$?
17. At the rate of $3\frac{3}{5}$ miles an hour, how long will I take to walk $45\frac{3}{10}$ miles?
18. A scarf requires $\frac{5}{8}$ of a yard of silk: how many scarfs can be made from $31\frac{1}{4}$ yards of silk?

19. How many bottles, each holding $1\frac{3}{8}$ pints, can be filled from a barrel of cider containing $61\frac{3}{4}$ pints, and how much will be left?

20. A man can run $\frac{1}{20}$ of a mile in $5\frac{1}{2}$ minutes, how far can he run per minute?

21. There are $2\frac{1}{4}$ inches in a piece of cloth: how many pieces would be the same length as $123\frac{3}{4}$ inches?

22. If I pay $\$\frac{1}{12}$ per pound for tea, how much tea can I buy for $\$9$? For $\$24$? For $\$64$?

23. If a man spend $\$4\frac{2}{5}$ a month on tobacco, in what time will he waste one week's wages, $\$27\frac{1}{2}$?

24. How long will $\frac{1}{13}$ of a cord of wood last a family using $\frac{3}{26}$ of $\frac{1}{2}$ a cord a day?

25. How long will a boy take to save $\$700$, if he earns $\$7\frac{1}{2}$ and spends $\$5\frac{3}{4}$ of it every week?

26. The "Chicora" steamed 156 miles in $10\frac{5}{6}$ hours: what is her speed per minute (60 minutes to the hour)?

---

168. Fractions are often written in this form

$$\frac{5\frac{1}{4}}{2\frac{5}{8}};\quad \frac{5}{6\frac{1}{4}};\quad \frac{\frac{1}{2}\text{ of }\frac{3}{4}}{\frac{2}{5}\text{ of }9};\quad \frac{\frac{2}{3}\times\frac{9}{10}\text{ of }\frac{3}{4}}{1\frac{1}{2}\text{ of }36}.$$

These fractions are called Complex Fractions.

169. A **Complex Fraction** is therefore a fraction which has a fraction in either its numerator or denominator, or in both.

170. Since we have seen that a fraction means the division of the numerator by the denominator, it follows that the above fractions can be simplified by ordinary division of fractions.

Thus:

Ex. 1.—$$\frac{5\frac{1}{4}}{2\frac{5}{8}}=\frac{21}{4}\div\frac{21}{8}=\frac{21}{4}\times\frac{8}{21}=2.$$

Ex. 2.—Simplify $\dfrac{\frac{2}{5}\times\frac{10}{12}}{4\frac{1}{3}\text{ of }\frac{2}{9}}$.

$$\frac{\frac{2}{5}\times\frac{10}{12}}{4\frac{1}{3}\text{ of }\frac{2}{9}}=\frac{\frac{1}{3}}{\frac{26}{27}}=\frac{1}{3}\div\frac{26}{27}=\frac{1}{\cancel{3}_{1}}\times\frac{\overset{9}{\cancel{27}}}{26}=\frac{9}{26}.$$

Ex. 3.—Simplify $(8\frac{1}{4}+3\frac{1}{2})\div 7\frac{3}{5}$.

☞ Since the expression $8\frac{1}{4}+3\frac{1}{2}$ is enclosed in the Bracket ( ), it must be simplified first, and the result divided by $7\frac{3}{5}$. If the bracket were omitted, we should first divide $3\frac{1}{2}$ by $7\frac{3}{5}$, and then add the quotient to $8\frac{1}{4}$, which would give a result quite different from the one required.

$8\frac{1}{4}+3\frac{1}{2}=11\frac{3}{4}=\frac{47}{4}$

$\frac{47}{4}\div 7\frac{3}{5}=\frac{47}{4}\times\frac{5}{38}=\frac{235}{152}=1\frac{83}{152}$

Ex. 4.—Simplify $2\frac{1}{5}+5\frac{1}{4}\times\frac{8}{21}-6\frac{2}{5}$ of $\frac{1}{2}+\frac{3}{11}\div\frac{5}{44}$.

$5\frac{1}{4}\times\frac{8}{21}=\frac{21}{4}\times\frac{8}{21}=2$

$6\frac{2}{5}$ of $\frac{1}{2}=\frac{32}{5}\times\frac{1}{2}=\frac{16}{5}$

$\frac{3}{11}\div\frac{5}{44}=\frac{3}{11}\times\frac{44}{5}=\frac{12}{5}$

$2\frac{1}{5}+2-\frac{16}{5}+\frac{12}{5}=4\frac{1}{5}-3\frac{1}{5}+2\frac{2}{5}=1+2\frac{2}{5}=3\frac{2}{5}$

☞ The pupil will carefully notice that unless a bracket interferes, the operations shown by the signs $\times$, **of**, and $\div$, are carried out before any of the others.

Thus: In Ex. 4, instead of adding $2\frac{1}{5}$ and $5\frac{1}{4}$, we multiply $5\frac{1}{4}$ by $\frac{8}{21}$, and instead of adding $\frac{1}{2}$ to $\frac{3}{11}$, we simplify $6\frac{2}{5}$ of $\frac{1}{2}$, and $\frac{3}{11}\div\frac{5}{44}$.

A bracket should always be used in any case of doubt.

171. A **Bar** or **Vinculum** has the same effect as a bracket.

The expression $\overline{\frac{3}{4}-\frac{5}{7}}$ of $\frac{28}{49}$ has the same meaning as $(\frac{3}{4}-\frac{5}{7})$ of $\frac{28}{49}$. The value of the expression as it now stands is $\frac{1}{49}$. If the vinculum or bracket had not been given, the result would be $\frac{67}{196}$, as the pupil may find.

EXERCISE 42.

Simplify the following expressions:

1. $\dfrac{\frac{3}{5}}{1\frac{1}{3}}$.

2. $\dfrac{\frac{3}{5}}{1\frac{3}{4}}$.

3. $\dfrac{\frac{5}{6}}{1\frac{2}{5}}$.

4. $\dfrac{1}{1\frac{1}{4}\times 1\frac{1}{2}}$.

5. $\dfrac{\frac{7}{8}}{1\frac{1}{3}}$.

6. $\dfrac{1}{1\frac{4}{5}\text{ of }1\frac{1}{2}}$.

7. $\dfrac{3\frac{1}{6}}{9\frac{1}{3}}$.

8. $\dfrac{3\frac{1}{2}\div 2\frac{1}{4}}{6\div 4\frac{1}{2}}$.

9. $\dfrac{\frac{7}{8}\text{ of }60\frac{8}{10}\times\frac{8}{85}}{\frac{8}{85}}$.

Simplify the following expressions:

10. $\dfrac{\frac{1}{8}\text{ of }\frac{5}{3}\text{ of }\frac{9}{11}}{5\times\frac{3}{8}\text{ of }\frac{6}{7}}$.

11. $\dfrac{8\frac{1}{4}\times\frac{1}{3}\text{ of }7}{\frac{3}{4}\text{ of }\frac{4}{3}\text{ of }5}$.

12. $26\times\frac{5}{7}+2\frac{1}{2}$.

13. $(4\frac{3}{7}-2\frac{1}{7})\times 3\frac{5}{7}$.

14. $2+6\frac{3}{7}\div\frac{5}{9}$.

15. $3\frac{1}{4}+(4\frac{3}{4}\div\frac{2}{3})$.

16. $4\frac{3}{4}\div(3\frac{1}{7}+\frac{2}{3})$.

17. $(14-8)\div\frac{3}{4}$.

18. $(3\div\frac{3}{4})-2\frac{1}{2}$.

19. $\dfrac{\frac{3}{8}\text{ of }\frac{4}{7}\text{ of }3\frac{2}{3}}{4\frac{2}{7}\div 8}$.

20. $(15-\frac{3}{5}-2\frac{1}{7}\times 5)\times 3\frac{1}{7}$.

21. $(4\frac{3}{4}+\frac{2}{3})\div(3\frac{1}{7}-1\frac{3}{14})\times 2$.

22. $\left(\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\dfrac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}\right)-\left(\dfrac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{4}+\frac{1}{6}}-\dfrac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}\right)$.

23. $\dfrac{\frac{3}{5}\text{ of }\frac{2}{9}-\frac{4}{7}\text{ of }\frac{1}{8}}{\frac{4}{15}\text{ of }\frac{1}{2\frac{1}{8}}+\frac{3}{10}\text{ of }\frac{7}{9}}$.

24. $\left(\dfrac{3\frac{3}{4}\times 3\frac{1}{3}}{2\frac{1}{2}+\frac{5}{18}}\div\dfrac{6\frac{2}{3}\times 8\frac{8}{9}}{8\frac{8}{9}-6\frac{2}{8}}\right)+\left(\dfrac{14\frac{2}{5}}{6\frac{6}{7}}\times\dfrac{5\frac{5}{9}}{7\frac{7}{11}}\right)$

GENERAL EXAMPLES ON FRACTIONS.

Ex. 1.—What part of 7 times 4 is one-ninth of 72? $7\times 4=28$; $\frac{1}{9}$ of $72=8$.

Now, what part of 28 is 8? Since 1 is $\frac{1}{28}$ of 28, 8 must be $\frac{8}{28}$ of it, that is $\frac{2}{7}$.

This question can be written in many ways. Instead of saying, "What part of 28 is 8?" we may say, "8 is what fraction of 28?" or "Express 8 as a part of 28," or, "What fraction is 8 of 28?"

Now the answer to each question is $\frac{8}{28}$. We thus see that we always place in the numerator the quantity which is to be the part or fraction of the other.

Ex. 2.—$4\frac{3}{5}$ is what part of $18\frac{2}{5}$?

Here we write $4\frac{3}{5}$ in the numerator and $18\frac{2}{5}$ in the denominator, and reduce the fraction thus formed to its lowest terms.

$$\frac{4\frac{3}{5}}{18\frac{2}{5}}=\frac{23}{5}\div\frac{92}{5}=\frac{\overset{1}{\cancel{23}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{92}}}=\frac{1}{4}$$

☞ The pupil should always prove that the result is correct. Thus:

Show that $\frac{1}{4}$ of $18\frac{2}{5}$ is $4\frac{3}{5}$.

$$\tfrac{1}{4} \text{ of } 18\tfrac{2}{5} = \frac{1}{4} \text{ of } \frac{92}{5} = \frac{23}{5} = 4\tfrac{3}{5}$$

Hence the answer $\frac{1}{4}$ is correct.

Ex. 3.—$\frac{4}{5}$ of 60 is $\frac{3}{8}$ of what?

Since $\frac{4}{5}$ of 60 = 48, the question now stands: "48 is $\frac{3}{8}$ of what?"

If 48 is $\frac{3}{8}$, then $\frac{1}{3}$ of 48, that is 16, must be $\frac{1}{8}$, and $\frac{8}{8}$, or the required number, must be $16 \times 8 = 128$.

PROOF: $\frac{3}{8}$ of $128 = 48$

Ex. 4.—$\frac{2}{3}$ of 27 is $\frac{6}{7}$ of how many times 3?

Since $\frac{2}{3}$ of 27 = 18, the question then stands: "18 is $\frac{6}{7}$ of what?"

This we know by Ex. 3 to be 21.

Now we must find how many times 3 this 21 is. We know it is 7 times, which is the required result.

PROOF: $7 \times 3 = 21$.

$\frac{6}{7}$ of 21 = 18.

EXERCISE 43.

1. What must be divided by $11\frac{1}{20}$ to produce $7\frac{4}{5}$?
2. To what must $\frac{8}{4\frac{1}{2}} + \frac{4\frac{1}{2}}{4}$ be added to give $\frac{3\frac{5}{7}}{4\frac{8}{11}} + \frac{9}{4}$?
3. What part of $\$5\frac{2}{5}$ is $\$1\frac{1}{2}$?
4. What part of a day (24 hours) is $5\frac{1}{3}$ hours?
5. $8\frac{4}{7}$ minutes is what fraction of an hour (60 minutes)?
6. What part of $\frac{5}{8}$ of a peck is $\frac{1}{2}$ a peck?
7. What part of \$11 is $\frac{3}{8}$ of \$16?
8. $\frac{3}{7}$ of a peck is $\frac{5}{8}$ of what?
9. $\frac{5}{11}$ of 132 pounds is $\frac{3}{4}$ of what?
10. 12 is $\frac{3}{4}$ of $\frac{5}{6}$ of what?

11. If \$120 was $\frac{6}{7}$ of what I paid for a horse, what was the price paid?
12. What is the cost of $2\frac{2}{11}$ loads at \$220 for 16 loads?
13. $\frac{6}{13}$ of 39 ounces is $\frac{9}{16}$ of the weight of a parcel: find its weight?
14. $\frac{3}{11}$ of $\frac{2}{3}$ of \$47 is $\frac{5}{7}$ of $\frac{5}{13}$ of a man's rent: find the rent.
15. A. had \$260, and spent $\frac{7}{13}$ of it, which was just $\frac{5}{11}$ of what B. earned: how much did B. earn?
16. Of how many times 9 is $\frac{3}{4}$ of 20 the five-ninths?
17. What number is $\frac{3}{5}$ of $4\frac{4}{9}$ times $\frac{3}{4}$ of 12?
18. What part of 42 is 3 times $\frac{2}{5}$ of 30?
19. I paid \$50 for a waggon, and $\frac{5}{5}$ of the cost of it was just $\frac{2}{3}$ of 3 times the cost of the harness: what was the harness worth?
20. If \$200 is 4 dollars more than $\frac{7}{5}$ of 4 times my money, what do I own?
21. Of how many times $\frac{2}{5}$ of \$10 is $\frac{3}{5}$ of \$25 the three-fourths?
22. If $\frac{4}{9}$ of 18 years is just $\frac{2}{9}$ of 4 times $\frac{3}{4}$ of a boy's age, what is his age?

Ex. 5.—A man owned $\frac{5}{12}$ of a vessel, and sold $\frac{1}{13}$ of his share: what part had he left?

If he sold $\frac{1}{13}$ of his share, he must have $\frac{12}{13}$ of his share left.

$$\frac{12}{13} \text{ of } \frac{5}{12} = \frac{5}{13}$$

☞ The pupil will likely proceed to find what part he sold, that is, $\frac{1}{13}$ of $\frac{5}{12}$, and then subtract the result from $\frac{6}{12}$, which, although correct, causes a waste of time, for we want to know, not the part sold, but the part left.

Ex. 6.—After giving $\frac{2}{7}$ of his estate to a college, a gentleman had \$55000 left: what had he at first?

If he gave away $\frac{2}{7}$ of the estate he must have $\frac{5}{7}$ of it left; and since $\frac{5}{7}$ = \$55000, then $\frac{1}{7}$ = \$11000, hence the whole or $\frac{7}{7}$ = \$77000.

23. A man gave away $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{15}$ of his money: what part had he left?

24. I owned $\frac{8}{9}$ of $\frac{7}{16}$ of $\frac{3}{10}$ of a business, and sold $\frac{1}{7}$ of my share: what part of the business do I now own?

25. A mine is worth $2200; a man owns $\frac{3}{11}$ of $\frac{2}{7}$ of it, and lost $\frac{5}{12}$ of his share: what part of the mine has he left, and what is he now worth?

26. A man owns $\frac{3}{8}$ of $\frac{5}{6}$ of $\frac{7}{10}$ of an investment; on selling $\frac{2}{7}$ of his share he finds himself worth $100 less than before: what is the value of the whole investment?

27. B. owned $(\frac{11}{12}-\frac{5}{9})$ of an estate, and sold $\frac{1}{3}$ of the estate: what part of the estate, and what part of his former share does he now own?

28. A man sold $\frac{1}{2}$ his load to the first one he met, $\frac{1}{3}$ of the remainder to the next one, $\frac{1}{4}$ of the remainder to the next one, and so on: what part of the load had he left after 10 bargains?

Ex. 7.—A field could be ploughed by A. in 4 days, and by B. in 5 days: in what time could both together do the work?

A. does the whole work in 4 days, therefore he does $\frac{1}{4}$ of the work in 1 day; also B. does $\frac{1}{5}$ of the work in 1 day. Both working together would thus do $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$ of the work in 1 day.

A. and B. do $\frac{9}{20}$ in 1 day.
" " $\frac{1}{20}$ in $\frac{1}{9}$ day.
" " $\frac{20}{20}$ or the whole work in $\frac{20}{9}=2\frac{2}{9}$ days.

Ex. 8.—Two pipes running together can fill a tank in 20 minutes: one of them could alone fill it in 35 minutes: how long would the other take to fill it?

Both pipes fill $\frac{1}{20}$ of the tank in 1 minute.
One pipe fills $\frac{1}{35}$ " " " "
The other must fill $\frac{1}{20}-\frac{1}{35}=\frac{3}{140}$ in 1 minute.
It fills $\frac{3}{140}$ of the tank in 1 minute.
" $\frac{1}{140}$ " " $\frac{1}{3}$ minute.
" $\frac{140}{140}$ " " $1\frac{40}{3}=46\frac{2}{3}$ minutes.

29. A. can do his work in $5\frac{1}{4}$ hours: how much can he do in an hour?

30. A. and B. can together do $\frac{3}{4}$ of a job in 6 days: how much can they do in a day?

31. A. can do the work in 8 days, B. in 3 days, and C. in 9 days: how long would the three together take?

32. A man can do $\frac{5}{30}$ of a journey in 1 day: how long will he take to finish it?

33. A pipe can fill $\frac{4}{17}$ of a vat in 3 hours: how long will it take to fill the whole?

34. A., B. and C. dig a ditch in 5 days; B. and C. can do it in 8 days: how long would A. take to do it?

35. A. can do a work in $\frac{1}{3}$ of a day, B. in $\frac{1}{6}$ of a day, and C. in 1 day: how long will the three men take?

36. A. mows 2 acres in 3 days, B. 5 acres in 6 days: in what time can they together mow 9 acres?

37. A., B. and C. can do a piece of work in a week; A. and B. do $\frac{5}{9}$ of it, B. and C. do $\frac{2}{3}$ of it: how much of it can B. do?

38. Three taps can fill a tank in 30 minutes, 40 minutes, and 60 minutes respectively: how long will they take, all being opened at the same time?

39. A pipe can fill a vat with water in 11 minutes; another can empty it in 20 minutes: in what time will the vat be filled if both are opened at the same time?

40. A man paid $\frac{1}{9}$ of his debts, then $\frac{1}{2}$ of the remainder: there was then due \$850: what did he owe at first?

41. A telegraph pole is $\frac{2}{5}$ of its length in air, $\frac{1}{5}$ in water, and the remainder, which is 8 feet, in the ground: what is the length of the pole?

42. A boy in flying his kite lost $\frac{3}{5}$ of the string, then added 65 feet, and found that it was just $\frac{5}{6}$ of the original length: what was the length at first?

CHAPTER IV.

CANADIAN MONEY.

172. The Government of every country makes or coins money, which is used by its people in buying and selling.

173. The Canadian Government coins and issues the following pieces of money: The **one cent piece**, made of copper; the **five cent piece, ten cent piece, twenty-five cent piece**, and the **fifty cent piece**, all of which are silver coins.

174.

The five cent piece	=	5 cents.
The ten cent piece	=	10 cents.
The twenty-five cent piece	=	25 cents.
The fifty cent piece	=	50 cents.

175. The Dollar, which is equal to 100 cents, is paper money, issued either by the Government or by the Banks of the country, and may be changed into coin at any time by presenting it at the Banks.

There are also Two-Dollar Bills, Four-Dollar Bills, Five-Dollar Bills, Ten-Dollar Bills, Twenty-Dollar Bills, Fifty-Dollar Bills, One Hundred-Dollar Bills, as the Banks or the Government may be pleased to issue.

176. As stated before, the Dollar is represented by the character $. Thus, $45 is read: Forty-five dollars.

The cents are often represented by the letter **c**, thus, 27c. is read: Twenty-seven cents.

If dollars and cents be taken together, they are written thus: $84.52 is read: Eighty-four dollars and fifty-two cents. $7.08 is read: Seven dollars and eight cents.

Since 25 cents = $\frac{1}{4}$ of a \$, \$30$\frac{1}{4}$ is the same as \$30.25.
50 cents = $\frac{1}{2}$ " \$17$\frac{1}{2}$ " \$17.50.
75 cents = $\frac{3}{4}$ " \$50$\frac{3}{4}$ " \$50.75.

177. We have seen in Arts. 81 and 91 how we change dollars into cents or cents into dollars.

178. The most necessary requirement in any business is the correct addition of sums of money expressed as dollars and cents.

Ex.—Add together \$30.65, \$150.75, \$80.73, \$712.40, \$212.04.

\$30.65
150.75
80.73
712.40
212.04
\$1186.57

The amounts are written as in Simple Addition. The sum of the cents is 257 cents, or 200 cents and 57 cents, that is, two dollars and fifty-seven cents. We write the 57 cents under the cents column, and carry the 2 dollars, and proceed to add as usual.

179. We see, then, that the Addition and Subtraction of Canadian money is the same as Simple Addition. Care, however, must be taken that the separating dot . be written in its proper place in the sum or difference.

180. We have now to multiply and divide sums of money by any number.

Ex.—Multiply \$241.35 by 8.

\$241.35
8
\$1930.80

Writing the numbers as usual, we first multiply the number of cents by 8, which gives 280 cents, or 2 dollars and 80 cents. Place the 80 cents in the usual place. Again, multiplying the number of dollars by 8, gives 1928 dollars, which with the two dollars carried from the cents, makes 1930 dollars. The final sum is therefore \$1930.80.

☞ This is the product that would have been obtained if the numbers had been multiplied as in Simple Multiplication.

181. To divide a sum of money by any number.

Ex.—Divide \$197.35 by 5; or, in other words, divide \$197.35 into 5 equal parts.

5) \$197.35
\$39.47

Writing the numbers as usual, we first divide the number of dollars. This gives 39 dollars and a remainder of 2 dollars, or 200 cents. The 200 cents taken with the 35 cents make up 235 cents, which being divided by 5 gives 47 cents.

Hence the quotient is \$39.47.

☞ This is the quotient that would have been found if the numbers had been divided as in Simple Division.

```
41 ) 5412 ( 132
     41
     ---
     131
     123
     ---
      82
      82
```

Ex.—How often will 41 cents be contained in \$54.12?

This is the same as finding how often 41 cents is contained in 5412 cents.

We therefore simply divide 5412 by 41, and obtain 132 times.

182. We thus see that Addition, Subtraction, Multiplication, and Division of Canadian Money is the same as in whole numbers.

☞ The statement in the last Article applies equally as well to United States money.

EXERCISE 44.

1. Add together \$34.12, \$43.24, \$21.34, \$20.41, \$34.12 and \$21.32.
2. I gave to A. \$41.23, to B. \$42.23, to C. \$14.43, to D. \$3.43, to E. \$24.02, to F. \$24.01: what did I give in all?
3. A man owes A. \$27.18, B. \$56.43, C. \$8.94, D. \$45.73, E. \$108.99, F. \$62.86, and G. \$5.09: how much does he owe in all?

4. Bought of Jas. Foster & Sons: Knives, $41.23; Nails, $40.32; Tools, $34.23; Fire-arms, $24.32; Gunpowder, $42.43; Sundries, $32.43: find the total cost.

5. I made the following deposits in the Dominion Bank: Cheques, $412.13; Notes, $401.21; Silver, $124.10; Gold, $132; Drafts, $301.24: find the total deposit.

6. Find the amount of the following accounts: $40.19, $78.51, $90.84, $112.79, $29.08, $5.18, $929.03, $33.33.

7. I spent $800.50 in Toronto, $75.40 more in Hamilton, but $101.05 less in London: what did I spend altogether?

8. Bought a farm for $3273.08, a house for $1503.91, horses for $429.17, cows for $273.54, sheep for $290.09, hogs for $447.26, and furniture for $298.98: what was the total amount?

9. Bought of Rice Lewis & Son the following bill: Tackle, $88.88; Rope, $99.99; Pulleys, $90.09; Wire, $7.70; Flags, $17.90; Steel, $183.84; and Cutlery, $611.12: find the amount of the bill.

10. After lending A. 60 dollars, B. 139 dollars 44 cents, C. 56 dollars 73 cents, D. 78 dollars 17 cents, E. $93.39, F. $45.09, I had left $357.28: what sum had I at first?

11. Add together 26 dollars 3 cents, 50 dollars 90 cents, 49 dollars 1 cent, 108 dollars 14 cents, 77 dollars 25 cents, 83 dollars 68 cents, and 40 dollars 8 cents.

12. From $593.15 take $208.28.

13. I owe a firm $503.01, and pay $267.08: what do I still owe?

14. What change do I receive from a hundred dollar bill if I pay for goods worth $67.43?

15. Find the difference between nine hundred and three dollars and twenty cents, and $705.82.

16. A. is worth $19327.30, and B. $29120.91: what is the one worth more than the other?

17. What is the cost of 327 oxen at twenty-seven dollars and three cents each?

18. Find the cost of 13 barrels of apples, each containing three bushels, at 73 cts. a bushel.
19. What will 50 boys receive for 2 days' work of 9 hours each, at the rate of 15 cts. an hour for each?
20. What will be the cost of 2192 pounds of lard at 9 cts. a pound?
21. Bought of the Virginia Tobacco Company, Toronto, 5 hogsheads of tobacco, each weighing 560 pounds, at 28 cts. a pound: what was the whole cost?
22. Find the value of the uniforms for 14 companies of 90 men each, at $\$41\frac{1}{4}$ per uniform?
23. A merchant sold 3725 barrels of flour, gaining 42 cts. on each barrel: find the total gain.
24. What will the paving of 880 square yards of road cost at $2.05 a square yard?
25. At 32 cts. a foot, what will 391 feet of rope cost?
26. Messrs. W. A. Murray & Co. received from Europe 10 bales of silk, each containing 27 pieces of 33 yards each: what was the value of the whole at $2.90 a yard?
27. How often will $85.14 contain 99 cents?
28. I buy books at $1.08 each, and pay $933.12: how many books do I buy?
29. Divide $18321 by $2.40.
30. If $221.20 be equally divided among 70 men, what does each get?
31. If $402.20 be equally divided among 19 men, how much will remain?
32. A bank pays off a debt of $147085.96 at the rate of $253.16 a month: how long will it take to pay it all off?
33. Bought from Eby, Blain & Co. 20 boxes of raisins, 22 pounds in each, and paid $66: how much is that a pound?
34. A dealer sells 15 bags of coffee for $378, worth 24 cts. a pound: how many pounds did he put in each bag?
35. Jones is worth $19381.37, and Brown $2917.39 less than Jones: what are both together worth?

36. A man having $379, bought 97 lambs at $2.95 each: how much had he left?

35. Sold J. Cleghorn & Son 37 boxes of figs at $2.75 a box, losing $27.30: what did they cost me at first?

38. Received from A. $19.89, from B. $33.24, from C. $25.47 more than from A.: how much less was received from B. than from C.?

39. Bought 359 shares at $1.20 a share, and had left $99.80: what sum had I at first?

40. Bought 324 pounds of tea for $243: if I sold it for 15c. a pound more than I gave for it, what was my whole gain and selling price per pound?

41. A man sold his house for $1567.30 and his land for $3121.30, and bought building lots at $23.80 each: how many could he buy?

42. While I was making $80 a week, I saved $98.35 in five weeks: what did I spend a week?

43. Borrowed from A. $93.86, $46.31, and $101.88; from B. $9.08; and then paid off my debt to C., which was $197.58: what had I left?

44. How many 25-cent pieces would pay for 80 horses at $102½ each?

45. A man sold 53 bags of flour at $1.04 a bag; his neighbor sold 13 bags less, but at 15 cts. more a bag: how much more did one get than the other?

BRITISH OR STERLING MONEY.

183. After understanding the coinage of our country, it is necessary to enquire into that of Great Britain, on account of the very close relation between the two countries.

184. The smallest coin of Great Britain is the Farthing, which is made of copper.

Four of these pieces make up in value one Penny, also a copper coin.

There are twelve pence in a Shilling (a silver coin), which passes in Canada for 24 cents.

Twenty shillings make the Pound, or gold Sovereign.

185. We thus have the following

TABLE OF STERLING MONEY.

4 farthings	make	1 penny.	d.
12 pence	"	1 shilling.	s.
20 shillings	"	1 pound.	£.

☞ The farthing being a quarter of a penny, is written $\frac{1}{4}$; two farthings, being the half of a penny, is written $\frac{2}{4}$; three farthings is written $\frac{3}{4}$.

Thus £7 8s. $7\frac{3}{4}$d. is read: 7 pounds, 8 shillings, 7 pence, 3 farthings.

186. We see that any number of pounds are brought to shillings by multiplying the number of pounds by 20; shillings to pence by multiplying by 12; pence to farthings by multiplying by 4.

Ex. 1.—Express £8 as pence.

£ 8 20	There are 20 shillings in £1, hence there must be 8 times 20 shillings in £8, that is 160 shillings.
160 shillings. 12	There are 12 pence in 1 shilling, hence there will be 160 times 12 pence in 160 shillings, that is 1920 pence.
1920 pence.	Therefore £8 = 1920 pence.

Ex. 2.—Express £2 10s. $11\frac{1}{2}$d. as farthings.

£ s. d. 2 10 $11\frac{1}{2}$ 20	In £2 there are 40 shillings, as before, but we must add to these the 10 shillings, which makes 50 shillings.
50 12	Again, in 50 shillings there are 600 pence, which when added to the 11 pence gives 611 pence.
611 4	Finally, in 611 pence there are 2444 farthings, which with the $\frac{1}{2}$d., or 2 farthings, makes 2446 farthings.
2446	Therefore £2 10s. $11\frac{1}{2}$d. = 2446 farthings.

187. In the same way, any number of a higher denomination may be brought to an equal value in a lower denomination.

188. We will now express any number of a lower **denomination** in units of a higher **denomination.**

Ex. 1.—Change 1200 farthings to pounds.

$$\begin{array}{r|l} 4 & 1200 \\ \hline 12 & 300 \\ \hline 20 & 25 \\ \hline & 1\ 5 \end{array}$$

Since 4 farthings make 1 penny, 1200 farthings will make just one-quarter of 1200 pence, that is 300 pence. In the same manner, 300 pence will make one-twelfth of 300 shillings, that is 25 shillings. Lastly, we divide the number of shillings by 20, to bring them to pounds, which gives 1 pound and 5 shillings remaining.

Hence 1200 farthings = £1 5s.

Ex. 2.—Express 3477 farthings in the higher orders.

$$\begin{array}{r|l} 4 & 3477 \\ \hline 12 & 869\frac{1}{4} \\ \hline 20 & 72\ 5\frac{1}{4} \\ \hline & 3\ 12\ 5\frac{1}{4} \end{array}$$

Dividing the farthings by 4. we obtain 869 pence and 1 farthing remaining. Dividing the 869 pence by 12, we find 72 shillings and 5 pence over, and finally, dividing the 72 shillings by 20 we have 3 pounds and 12 shillings over.

Therefore 3477 farthings = £3 12s. $5\frac{1}{4}$d.

189. In the same way, any number of units of a lower order may be brought to the same value in units of a higher order.

EXERCISE 45.

1. How many farthings in $14\frac{1}{2}$d.? In 273d.?
2. How many pence in 468s.? In £55 19s. 7d.?
3. Express £754 17s. $9\frac{3}{4}$d. in farthings.
4. How many pounds in 7660s.? In 114720d.?
5. Reduce £15 8s. $7\frac{1}{2}$d. to farthings
6. Change 21368 farthings to pounds, etc.
7. Reduce 854d. to pounds, etc.
8. Bring £3 19s. 7d. to farthings.
9. Express 4s. $2\frac{3}{4}$d. in farthings.
10. Reduce £21 0s. $0\frac{3}{4}$d. to farthings.
11. Express in £ s. d.:
    (1) 5317 pence. (2) 16629 farthings.
    (3) 38003 farthings. (4) 720370 pence.

12. How many six-penny pieces in £528 6s. 6d. ?

13. How many pounds, etc., in 3729 three-penny pieces ?

14. In an English school, half the boys wrote in copy books, and paid 3d. each week ; 99 boys paid 2d. a a week ; and 59 paid 1d. : how many in the school, and what was paid in all?

---

190. To add together any sums of money.

Ex. 1.—Add together £4 8s. 6¼d., £1 9s. 11¾d., 10s. 4½d., £3 19s. 11¾d.

£	s.	d.
4	8	6¼
1	9	11¾
	10	4½
3	19	11¾
£10	8	10¼

We first place the quantities so that the same kind are in the same column.

Adding the farthings' column, we get 9 farthings, that is 2 pence and 1 farthing. We write the 1 farthing in its proper place, and carry the 2 pence to the pence column.

The pence column gives when added 32 pence, which with the 2 pence carried is 34 pence, or 2 shillings and 10 pence.

Write the 10 pence under the proper column, and carry the 2 shillings to the sum of the shillings' column.

This makes in all 48 shillings, which is equal to 2 pounds and 8 shillings. Place the 8 shillings under the proper column, and carry the 2 pounds.

This, when added to the pounds' column, gives 10 pounds.

Hence the total sum is £10 8s. 10¼d.

☞ As in Art. 51 of Simple Addition, we might have placed each total under its column, as £8 48s. 34¾d., but, as in that case, we always change the units of the lower order into units of a higher, and write down the remaining units under their own column.

EXERCISE 46.

Add together the following sums of money:

1. £15 10s. 9d., £8 9s. 7d., £1 12s. 10d., £1 18s. 4d.

2. £8 9s. 7¾d., £7 12s. 4¼d., £1 19s. 11½d., 15s. 8¾d.

3. £9 8s. 10d., £8 16s. 11d., £7 8s. 3d., £8 16s. 2d., £7 3s. 4d., £8 17s. 2d., £3 8s. 11d., £6 9s. 2d., £3 7s. 5d.

4. £8 17s. 5d., £5 8s. 6¼d., £7 4s. 4½d., 19s. 4¾d., £18 10s. 11d., £3 7s. 4d., £5 12s. 7¾d., £8 19s. 2d., £7 2s. 4d.

5. £94 15s. 5¾d., £87 16s. 6½d., £91 17s. 7½d., £67 18s. 8¾d., £84 19s. 9½d., £98 os. o¾d., £56 17s. 11d., £133 3s. 10¼d., £212 18s. 9d.

6. £816 17s. 8½d., £389 18s 10¼d., £31 17s. 11d., £346 18s. 6½d., £407 13s. 8¾d., £748 11s. 11d., £567 14s. 4¾d., £687 15s. 10¼d., £827 16s. 10¾d.

7. £7148 11s. 8½d.+£3596 18s. 11¼d.+£71416 13s. 8½d. +£81 11s. 4d.+£7186 13s. 4¾d.+£714 13s. 8¾d.+ £8196 18s. 10½d+£811 8s. 6d.

8. £17846 17s. 8d.+£3479 13s. 11d.+6783 14s. 5d.+ £687 15s. 10d.+£8412 11s. 4d.+£6791 15s. 7d.+ £6149 17s. 8d.+£8416 11s. 3d.+£879 18s. 4d.+ £7358 13s. 8d.

9. £4738 17s. 2d.+£3947 19s. 8d.+£7135 13s. od.+ £914 os. 8d.+£4783 15s. 11d.+£7198 17s. od.+ £8359 11s. 8d.+£8746 os. od.+£879 8s. 7d.+£9157 16s. 8d.

10. £3109 os. 11d.+£798 13s. 4½d+£9146 13s. 7d.+ £874 os. 8d.+£9146 3s. 4d.+£8749 13s. 5d.+£8735 19s. 9d.+£9146 17s. 8d.+£874 13s. 4½d.+£68 10s. 4¾d.

191. To subtract one sum of money from another.

Ex. 2.—Take £3 5s. 8¾d. from £5 4s. 6½d.

£	s.	d.
5	4	6½
3	5	8¾
£1	18	9¾

Setting down the quantities as usual, we say, 3 farthings from 2 farthings we cannot. Add one of the next higher order, that is, 1 penny or 4 farthings, which makes 6 farthings. Then 3 farthings from 6 farthings leaves 3

farthings. Again, having taken one of the 6 pence, we have to take 8 pence from 5 pence, which we cannot. Add 1 shilling or 12 pence. Then 8 pence from 17 pence leaves 9 pence.

Again, having used one of the 4 shillings, we have to take 5 shillings from 3 shillings, which we cannot. Add 1 pound or 20 shillings.

Then 5 shillings from 23 shillings leaves 18 shillings.

Lastly, having used one of the 5 pounds, we have to take 3 from 4 pounds, which leaves one pound.

Hence the difference is £1 18s. $9\frac{3}{4}$d.

☞ The only difference between this and simple subtraction is that, in simple subtraction, we always add 10, because 10 units of any order make 1 of the next higher, while in the present case the number of units in the orders above differs in each, that is, 4 in the first, 12 in the second, and 20 in the third. In every other respect the operations are exactly alike.

EXERCISE 47.

1. From £58 19s. $9\frac{1}{4}$d. take £50 2s. $4\frac{1}{4}$d.
2. How much greater is £60 8s. 9d. than £50 19s. 11d?
3. I owned £715 10s., and lost £620 14s. $6\frac{1}{4}$d.: what am I now worth?
4. Subtract £92004 18s. $5\frac{3}{4}$d. from £99153 10s. $2\frac{1}{4}$d.
5. A man had £8 6s. $9\frac{3}{4}$d.; he gave away £2 4s. $5\frac{1}{4}$d., and then spent £3 0s. 4d.: what had he left?
6. Bought goods for £9 6s. $10\frac{1}{4}$d., and paid £2 17s, $2\frac{3}{4}$d., what do I still owe?
7. What must be put with £47541 5s. $6\frac{3}{4}$d. to make up £50650 6s. $5\frac{3}{4}$d.?
8. A man has in cash £3443 15s., a house worth £474 8s. 9d., goods worth £713 11s., a farm worth £574, debts due to him £315, a vessel valued at £957 18s. $11\frac{1}{2}$d; but he owes A. £115 7s. 8d., B. £74 13s. 4d., and C. £327 18s. $4\frac{3}{4}$d: find what he is really worth.
9. From six hundred and seventy-one pounds eleven shillings and eight pence, take the sum of £600 15s. and twenty-five pounds and five pence.

192. To multiply any sum of money by a whole number.

Ex. 3.—What will £12 10s. 8¼d. produce when repeated 11 times?

£	s.	d.
12	10	8¼
		11
£132	110	88¼

Set down the numbers as usual. We will, in the first place, leave each product unchanged under its own column or order, as in Art. 190, thus: Eleven times 1 farthing is 11 farthings; 11 times 8 pence is 88 pence; 11 times 10 shillings is 110 shillings; 11 times 12 pounds is 132 pounds.

The result is £132 110s. 88¼d. Now 11 farthings are 2 pence and 3 farthings. We set down the 3 farthings and carry the 2 pence to the 88 pence, making it 90 pence. Again, 90 pence make 7 shillings and 6 pence. Place the 6 pence under its own column, and carry the 7 shillings to the 110 shillings, making it 117 shillings. But 117 shillings are 5 pounds and 17 shillings. Place the 17 shillings as usual, and carry the 5 pounds to the 132 pounds, making 137 pounds. The work would then appear thus:

£	s.	d.
12	10	8¼
		11
£137	17	6¾

193. This will be seen to agree perfectly with simple multiplication of numbers, and every remark made in regard to simple multiplication will apply to the present case.

194. We may, therefore, use the factors of any multiplier, instead of taking the multiplier at once.

Ex. 4.—If 24 men earn £3 15s. 10½d. each, what is earned in all?—that is, find 24 times £3 15s. 10½d.

£3	15	10½
		3
11	7	7½
		8
£91	1	0

The factors of 24 are 3 and 8. First multiply by 3, which gives 11 pounds 7 shillings and 7 pence half-penny, or 2 farthings. Next multiply by 8, as follows: 8 times 2 farthings = 16 farthings = 4d. 8 times 7 pence = 56 pence. This with the 4d. carried

gives 60 pence, which = 5 shillings and no pence remaining. 8 times 7 shillings = 56 shillings. This, with the 5s. carried, makes 61s., or £3 and 1s. 8 times £11 = £88, which, with the £3 carried, gives £91.

Hence the amount earned is £91 1s.

EXERCISE 48.

1. What is the value of three articles at £4 6s. 7½d. each?
2. One man can earn £9 8s. 4¼d.: what will 4 men earn?
3. At 17s. 3¾d. each, what must I pay for 7 articles?
4. What does £74 18s. 11½d. produce when repeated 6 times?
5. Find 8 times £18 os. 11d.
6. One-ninth of a vessel is worth £17 15s. o¼d.: what is the whole worth?
7. What will 10 persons spend at the rate of £13 5s. 7¼d. each?
8. Find the value of 11 sewing-machines at £18 os. 4¼d. each.
9. Multiply £70 os. 11½d. by 12.
10. If 18 men each subscribe sixty-three pounds and eight pence three farthings, what will the whole subscription amount to?
11. At 1d. each, articles cost £13 17s. o½d., what will they cost at 3s. each?
12. 29 men are employed on a road: what will be required to pay them at the rate of £5 7s. 6½d. each?
13. What amount must be divided among 35 men that each may receive £132 6s. 3¼d.?
14. A drover makes a profit of £2 9s. 2¼d. on every animal he takes to market: what will he gain in a day when he takes 31 to market?
15. Of what amount is £81 os. 8½d. the 187th part?

16. The "Dominion" carried to Liverpool from Quebec 132 head of cattle; on arrival they sold for £8 7s. 9½d. per head: what did the owner receive for them?

17. Brick is worth in England £2 5s. 7d. a thousand: what will be the cost of brick to build a bake-house requiring 10 thousand?

18. What must be paid for 12 dozen Christmas geese at 7s. 2¾d. each?

---

195. To divide any sum of money by a whole number.

When the divisor is 12 or under, the work is carried on mentally, as in Simple Division.

Ex. 5.—If £812 15s. o¼d. be divided among 11 women, what will each receive? or, divide £812 15s. o¼d. by 11.

£	s.	d.
11)812	15	o¼
73	17	8¾

Set down the numbers as usual. As in simple division, 11 is contained in 812, 73 times and 9 pounds over, that is, the eleventh part of £812 is £73 and £9 remaining. Write the £73 in its proper place, and carry the £9.

Now the £9 must be brought to shillings, and then added to the other 15 shillings, that is, 180+15=195. Again, 11 is contained in 195, 17 times and 8 shillings over. Write the 17s. in its proper place, and carry the 8 shillings.

These 8 shillings are equal to 96 pence, and as there are no pence to be added, 11 is contained in 96, 8 times and 8d. over. Write the 8d. as usual, and carry the 8d. left over.

These 8 pence, with the 1 farthing, make 33 farthings, and 11 is contained in 33, 3 times. Write down the 3 farthings in its place.

Each woman, therefore, would receive £73 17s. 8¾d.

196. When the divisor is greater than 12.

Ex.. 6.—Divide £93 16s. 10½d. into 99 equal parts.

£ s. d. s. d.
99) 93 16 10½ (18 11½
20
1860
16 (added)
)1876 (18
99
886
792
94
12
1128
10 (added)
)1138 (11
1089
49
4
196
2 (added)
198 (2
198

Here, £93 will not contain 99, hence it is reduced to shillings (1860), and the 16s. added, making 1876 shillings. This contains 99, 18 times. Place this 18s. in the quotient, and bring the remaining 94s. to pence (1128), add in the 10d., and divide the result, 1138 pence, by 99. This gives 11 pence, and 49 pence over. Write the 11d. in the quotient. Bring the 49 pence to farthings (196), and add the ½d., or 2 farthings, making 198 farthings. Finally, divide by 99, and we have 2 farthings, or ½d., to place in the quotient.

Thus the quotient is 18s. 11½d.

☞ The above agrees exactly with the work performed in Art. 100, the multipliers there, however, being 10 in each case, instead of 20, 12, and 4, as in the present case.

197. To divide one sum of money by another.

Ex. 7.—If each person earn 17s. 11¾d., how many persons will £50 6s. 10d. pay? In other words, how many times does £50 6s. 10d. contain 17s. 11¾d.?

Before we can divide one number by another, we have shown that they must be of the same kind. Hence we must bring each of the given amounts to units of the same order, that is farthings.

£50 6s. 10d. = 48328 farthings
17s. 11¾d. = 863 "

The question now is: How often are 863 farthings

contained in 48328 farthings. This will be found by division.

803) 48328 (56
4315
5178
5178

Hence there will be 56 persons.

198. From the above we obtain the following

RULE.

To divide one amount by another, reduce each to the lowest order mentioned in either. Then divide as in simple division.

☞ In the preceding examples we have made use of **Sterling Money** to explain the addition, subtraction, multiplication and division of compound quantities. The same principles apply to all other compound quantities.

EXERCISE 49.

1. Find the fourth part of £57 18s. 9d.
2. How much more in the fifth part of £798 13s. 4d. than in the sixth part of £33 17s. 6d.?
3. What is the value of a sixth share in a vessel that cost £899 7s. 6d.?
4. Divide £812 15s. 0¼d. into 11 equal parts.
5. If 23 men earn £91 11s. 4½d., what is that apiece?
6. I paid £92961 11s. 1¼d. for 111 lots of land: how much was that for each?
7. The cost of carrying 149 cattle to Liverpool by the "Allan Line" was £589 9s. 7½d.: how much was that a head?
8. What sum of money must be multiplied by 9027 to produce £7175571 14s. 0¾d.?

9. Divide £6917 7s. 1½d. by 129.
£4747 5s. 3¾d. by 375.
£17303 12s. 11d. by 250.
£7957 4s. 6½d. by 109.

10. If 14 hundred-weight of sugar cost in Bristol £35 8s. 9d., how much is that per hundred-weight?

11. If 99 bags of figs cost in Glasgow £413 18s. 10½d., what will a bag cost in Edinburgh, the carriage being 6s. 4½d. per bag?

12. Divide £3 7s. 4½d. by 9s. 7½d.
£803 7s. 4d. by £73 os. 8d.
£255 3s. od. by £15 18s. 11¼d.
£907 4s. od. by £8 8s. od.

13. To how many men will £37 3s. allow £4 12s. 10½d. each?

14. A lady spent £75 os. 9d. in lace shawls at £6 5s. o¾d. each: how many shawls did she buy?

15. What part of £670 7s. is £83 15s. 10½d.?

16. How often is £280 15s, o½d. contained in £33971 os. o½d.?

17. How many purses of £12 7s. 10¼d. each can be made up of £14871 5s.?

18. How many times can £13 os. o¼d. be taken from £175 3s. 6¾d., and how much will be left?

---

199. After the Table of Money, the most necessary, perhaps, is the Table of Weight, as nearly all the articles of merchandise are bought and sold by weight.

200. The smallest weight used for this purpose, in practice, is the ounce, sixteen of which make up a pound.

201. The Table in full is as follows:

AVOIRDUPOIS WEIGHT.

16 ounces (oz.) make 1 pound, lb.
100 pounds " 1 hundredweight, cwt.
20 cwt. " 1 ton, T.

It is used in weighing all heavy merchandise, such as iron, groceries, etc.

☞ The pupil should be taught to express every order in units of every lower order; for example

1 T. = 20 cwt. = 2000 lbs. = 32000 oz.
1 cwt. = 100 lbs. = 1600 oz.
1 lb. = 16 oz.

A similar table may be made for every other weight or measure.

202. A different table is used when dispensing drugs, medicines etc., and is called

APOTHECARIES' WEIGHT.

20 grains (gr.) make 1 scruple, sc.
3 scruples " 1 dram, dr.
8 drams " 1 ounce, oz.
12 ounces " 1 pound, lb.

203. The following is the table used by jewellers in weighing precious stones, gold, silver, etc., and is called

TROY WEIGHT.

24 grains (gr.) make 1 pennyweight, dwt.
20 pennyweights " 1 ounce, oz.
12 ounces " 1 pound, lb.

☞ The "grain," "ounce" and "pound" are the same in the two latter tables, that is, a grain of gold dust would balance a grain of quinine.

204. All examples in the preceding tables are worked just as in the case of sterling money.

Ex. 1.—In 2 lbs. 5 dwt. of gold, how many grains?

lbs.	dwt.	
2	5	Since there are 12 oz. in 1 lb., there must
12		be 12 times 2 or 24 oz. in 2 lbs.
24 oz.		As there were no ounces given, we have
20		none to add to the 24 ounces.
485 dwt.		Again; since there are 20 dwt. in 1 oz.,
24		there must be 24 times 20, or 480 dwt.
1940		in 24 ozs. This, with the 5 dwt. given,
970		makes a total of 485 dwt. In the same
11640 grs.		manner, 485 dwt. equals 485 times 24, or 11640 grains.

Ex. 2.—A quantity of sugar weighing 11 T. 6 cwt. 10 lbs., just fills 5 vats; what will each vat hold?

Here we must find the fifth part of the given weight.

T.	cwt.	lbs.	oz.
5)11	6	13	0
2	5	22	$9\frac{3}{5}$

Dividing 11 tons by 5, we get 2 tons and 1 ton over.
1 T. 20 cwt., which, with the given 6 cwt., makes 26 cwt.
26 cwt. ÷ 5 = 5 cwt. and 1 cwt., or 100 lbs. over. This, with the 13 lbs., makes 113 lbs.
113 lbs. ÷ 5 = 22 lbs. and 3 lbs., or 48 ounces over.
48 oz. ÷ 5 = $9\frac{3}{5}$ ounces.
Each vat will thus hold 2 T. 5 cwt. 22 lbs. $9\frac{3}{5}$ oz.

EXERCISE 50.

The first eleven examples to be solved mentally.

(*a*) 1. What part of a pound of tea is 4 oz.? 8 oz.? 12 oz.?

2. What part of a ton is 5 cwt.? 12 cwt.? 8 cwt.?

3. What part of a cwt. is 25 lbs.? 50 lbs.? 75 lbs.?

4. What is the difference between 1 grain and 1 pennyweight? Between 1 ton and 1 cwt.? Between 1 dram and 2 scruples? Between 2 oz. and 12 drams? Between 1 hundredweight and 60 pounds?

5. How many grains of silver in $\frac{1}{6}$ of a lb.? In $\frac{1}{4}$ of a lb.? In $\frac{1}{8}$ of a lb.? In $\frac{3}{4}$ of a lb.?

6. How many grains in 1 pound of old silver?

7. How many ounces in 3 pounds of ginger? In 2 lbs. of quinine? In $\frac{3}{4}$ of a lb. of sago? In 1 lb. 5 oz. of old silver?

8. How many ounces in $\frac{1}{4}$ lb. of tea, $\frac{1}{2}$ lb. of sugar, and $\frac{1}{8}$ of a lb. of pepper?

9. What is the cost of $\frac{1}{2}$ lb. of nutmegs at 8 cents an ounce? Of $\frac{4}{5}$ of a scruple of quinine at 3 cents a grain? Of $\frac{2}{3}$ of a lb. of gold coin at $20 an ounce?

10. How many ounces in $\frac{1}{2}$ a cwt.?

11. Which is the cheaper of the following prices:

(*a*) ½d. a grain or 1s. a dwt.?
(*b*) $5 a cwt. or $100 a ton?
(*c*) 20c. an oz. or $2.50 a lb. of opium?
(*d*) 48c. a lb. or 4c. an oz. of spice?
(*e*) 50c. a lb. or $60 a cwt.?
(*f*) $2 a dwt. or $50 an oz.?

(*b*) 1. Write out a Table for Troy and Apothecaries' Weight like the Table in Art. 201.

2. In 1746 grains Troy, how many oz., etc.?

3. In 5 lbs. 7 dwt. how many grains?

4. How many pounds, etc., in one million grains Troy?

5. Find the number of grains in 10 lbs. 1 oz. 10 dwt. 1 gr.

6. How many grains of calomel in 7 oz. 3 sc.?

7. Find the number of lbs. in 691 sc.

8. How many powders of morphine of 1 grain each can be made from 3 lbs. 2 oz. 3 dr. 2 sc. 5 grs.?

9. In 5 cwt. 84 lbs. of sugar, how many one pound packages?

10. How many ounces of 80-cent tea can be put up from 3 cwt. 55 lbs. 4 oz.?

11. How many cwts., etc., make 5767 oz.?

12. Change 7359 pounds of coal to tons, etc.

13. A dealer sold to one customer 3 tons 5 cwt. 17 lbs. 13 oz. of sugar; to another, 4 tons 7 cwt. 35 lbs. 12 oz.; to another 1 ton 15 cwt. 63 lbs. 7 oz.: how much sugar did he sell in all?

14. What is the sum of 15 tons 6 cwt. 45 lbs. 5 oz.; 3 tons, 17 cwt. 80 lbs. 6 oz.; and 26 tons 31 lbs. 7 oz.?

15. What is the sum of 21 lbs. 7 oz. 12 dwts. 10 grs.; 28 lbs. 5 oz. 8 dwts. 7 grs.; 7 lbs. 6 dwts. 15 grs.; 41 lbs. 6 oz. 20 grs.; and 9 lbs. 7 grs.?

16. From 16 cwt. 90 lbs., take 8 cwt. 58 lbs. 6 oz.

17. From 85 tons 16 cwt. 39 lbs., take 61 tons 14 cwt. 68 lbs.

18. A man sold 15 loads of grain, each weighing 1 T. 270$\frac{1}{3}$ lbs; what did the whole weigh?

19. If 25 men each buy 16 T. 3 cwt. 10$\frac{1}{5}$ lbs. of goods, what do they buy in all?

20. Each of 56 goblets weighs 12 lbs. 3 oz. 16 dwt.; find the weight of the whole.

21. If a Mexican dollar weigh 17 dwt. 4$\frac{1}{2}$ grs., what will 96 dollars weigh?

22. A grocer buys 13 hogsheads of sugar weighing 6 T. 8 cwt. 57 lbs., what did each weigh?

23. If 31 cwt. 18 lbs. of rice be put up in parcels of 3 lbs. 8 oz. each, how many parcels will there be?

24. How many ounces will be left over?

25. How many bags of salt, each containing 2 cwt. 35 lbs., are there in 3 T. 3 cwt. 45 lbs?

26. How many forks, each weighing 2 oz. 10 dwt. can be made from 13 lbs. 7 oz. 15 dwt. of silver, and what will be left over?

27. A grocer having 17 cwt. 69 lbs. of soda, sold 4 cwt. 96 lbs. of it, and the remainder he put into six boxes; how much did each box contain?

28. Divide 7 lbs. 6 oz. 13 dwt. by 3 oz. 15 dwt. 13 grs.

29. Divide 131 lbs. 2 oz. 15 dwt. 20 grs. by 2 lbs. 7 oz. 9 dwt. 22 grs.

30. Divide 3 T. 2 cwt. 40 lbs. by 12 cwt. 48 lbs.

31. If one-thirteenth of a certain gold coinage be alloy, what is the quantity of pure gold in 274 pieces weighing 54 grs. each?

32. What is the total weight of silver in half a dozen dishes, each weighing 49 oz. 3 dwt. 4 grs.; a dozen plates, each weighing 16 oz. 17 dwt.; and a salver, weighing 126 oz. 15 dwt. 18 grs.?

33. Add together $\frac{2}{3}$ of 5 cwt. 40 lbs.; $\frac{6}{7}$ of 3 tons 6 cwt. 10 lbs.; and $\frac{5}{8}$ of 50 lbs.

34. What is the difference between $\frac{1}{3}$ of $4\frac{1}{2}$ of 2 lbs. 5 oz. 6 dwt.; and $\frac{3}{7}$ of $2\frac{1}{3}$ of 6 oz. 10 dwt. 10 grs.?

35. How often is $\frac{3}{7}$ of 5 scr. of quinine contained in a package containing 6 lbs. 7 oz.?

36. What weight is that of which 17 lbs. 2 oz. is $\frac{1}{16}$? of which $5\frac{1}{2}$ cwt. is $\frac{9}{11}$?

37. Of what weight is 2 oz. 3 dwt. three-seventeenths?

38. What part of 3 cwt. 6 lbs. would just balance 2 lbs. $6\frac{1}{2}$ oz.?

39. Add together $\frac{1}{3}$ of 2 lb., $\frac{3}{7}$ of 5 oz., $6\frac{7}{8}$ dwt., and $3\frac{1}{3}$ grs.

40. By how much does the $\frac{1}{4}$ of $6\frac{1}{2}$ tons exceed the $\frac{1}{3}$ of 13 cwt. 17 lbs. $5\frac{1}{4}$ oz.?

41. How much weight must be added to $3\frac{4}{11}$ cwt. to make 1 ton 2 cwt. 12 lbs.? and what weight taken from $23\frac{5}{16}$ tons will leave $\frac{5}{7}$ of $9\frac{1}{4}$ lbs.?

42. Take from 5 tons of potatoes its third, its fourth, and its fifth part; what part of $17\frac{1}{4}$ tons is the remainder?

---

205. All distances, lengths, breadths or widths, and heights, are expressed in miles, yards, feet, inches, &c.

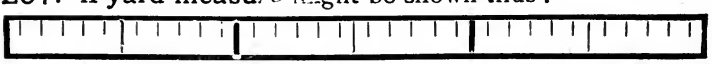
Thus: Toronto is distant from Montreal 333 miles; a room is 20 feet long; a piece of cloth is 22 inches wide; a flag pole is 80 feet in height.

206. These are included in a table called

LINEAR, or LONG MEASURE.

12 inches (in.)	make	1 foot, ft.
3 feet	"	1 yard, yd.
$5\frac{1}{2}$ yards	"	1 rod, rd.
40 rods	"	1 furlong, fur.
8 furlongs	"	1 mile, mi.

207. A yard measure might be shown thus:



The three larger divisions would represent **one foot** each.

The thirty-six smaller divisions would each represent **one inch.**

208. Cloth, ribbons, etc., are usually sold by this measure, the following parts of the yard being the most common:

Half of a yard, = 18 inches.
Quarter of a yard, = 9 "
Eighth of a yard, = $4\frac{1}{2}$ "
Sixteenth of a yard, = $2\frac{1}{4}$ "

☞ The sixteenth of a yard was formerly known as a "**nail.**"

The Flemish Ell was $\frac{3}{4}$ of a yard, or 27 inches.
" English Ell " $\frac{5}{4}$ " or 45 inches.
" French Ell " $\frac{6}{4}$ " or 54 inches.

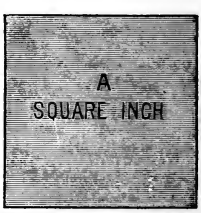
These Cloth Measures are now seldom used.

☞ The following measures are used for special objects:

1 Hand = 4 in., for measuring the height of horses.
1 Fathom = 6 ft., for measuring the depth of water.
1 Chain = 100 links = 66 ft., for measuring roads and surveying lands.
80 chains = 1 mile.

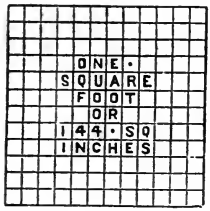
209. By Long Measure we would find the length and breadth of a room to be a certain number of feet, yards, etc. We now wish to measure the surface of the floor.

210. If a piece of paper were 1 inch long and 1 inch broad, it would represent what is called 1 **Square Inch** of **Area** or **Surface.**



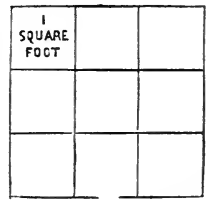
211. A Square Inch is, therefore, a square, one inch in length and one inch in breadth.

212.



In the same way, a Square Foot is a square, each side of which is one foot, or 12 inches in length, and from the figure we see it must contain 144 square inches.

A Square Yard will thus contain 9 square feet, as seen from the figure where each of the small squares represents a sq. foot.



213. These are all included in the following table, called

SQUARE MEASURE.

144 square inches (sq. in.)	make	1 square foot, sq. ft.
9 square feet	"	1 square yard, sq. yd.
$30\frac{1}{4}$ square yards	"	1 square rod, sq. rd.
160 square rods	"	1 acre, ac.
640 acres	"	1 square mile.

☞ The pupil will see that the area of the figures in Arts. 211 and 212 were found by multiplying their length by their breadth, thus:

1 square foot $= 12 \times 12 = 144$ sq. in.
1 square yard $= 3 \times 3 = 9$ sq. ft.
also, 1 square rod $= 5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$ sq. yds.

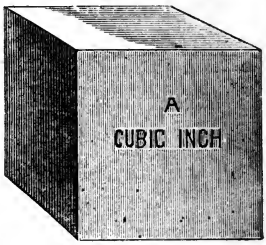
From this we see that part of the above table is derived from the table of Long Measure, the 12 in., 3 ft. and $5\frac{1}{2}$ yds occurring there first.

214. We see then that to find the area of a floor, etc., we multiply its length by its width.

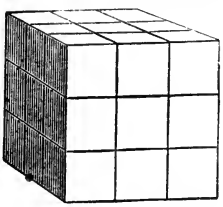
Ex. A table 10 feet long and 5 feet wide contains 50 square feet; for if it were only 1 foot wide, it would contain 10 square feet, but, being 5 feet wide, it must contain 5 times 10 sq. feet, or 50 sq. ft.

215. The only other measurement required is that of a solid block of marble, wood, etc., knowing its length, breadth and thickness.

216. If a block of wood were 1 inch long, 1 inch wide and 1 inch thick, it would represent what is called a **Cubic Inch.**



217. If the square foot of surface in Fig. 2, Art. 212 had been 1 in. thick, it would contain 144 cubic inches, and, therefore, if it had been 1 foot, or 12 inches thick, it would contain 12×144, or 1728 cubic inches, and be what is known as a **Cubic Foot.**



In the same manner, if the square yard of surface in Fig. 2, Art. 212 had been one yard or three feet thick, it would contain 3×9, or 27 cubic feet, and represent a **Cubic Yard.**

218. We thus make up the table of

CUBIC or SOLID MEASURE.

1728 cubic inches (cu. in.) make 1 cubic foot, cu. ft.
27 cubic feet " 1 cubic yard, cu. yd.

☞ This table, like the last, may be derived from that of Long Measure, thus:

$12 \times 12 \times 12 = 1728$ cubic inches in 1 cu. ft.
$3 \times 3 \times 3 = 27$ cubic feet in 1 cu. yd.

219. We see, then, that to find the contents of a solid, we multiply its length, breadth and thickness together.

Ex. Find the contents of a block of stone, 2 yds. long, 4 feet wide, and 3 feet deep.

2 yards long = 6 feet long.

$6 \times 4 = 24$ sq. ft. = its top surface.

Now, a slab from the top, 1 foot thick, would contain 24 cubic feet.

Therefore, being 3 feet deep, there would be 3 times 24 cubic feet = 72 cubic feet = 2 cu. yds., 18 cu. ft.

☞ A cord of wood is 8 feet long, 4 feet high, and 4 feet wide; or 128 cubic feet.

Ex. Express 7544 yards as miles, etc.

$5\frac{1}{2})7544$
2 2

11)15088

4 m 0)137 r 1—7

8)34—11

4—2

First, we reduce the yards to rods by dividing $5\frac{1}{2}$, thus:

Bring the $5\frac{1}{2}$ yards to half-yards, that is 11; then bring the 7544 yards to half-yards, that is 15088: then 11 half-yards are contained in 15088 half-yards, 1371 times and 7 half-yards, or $3\frac{1}{2}$ yards remaining.

Proceeding, as usual, we obtain 4 mi. 2 fur. 11 rds. $3\frac{1}{2}$ yds., which may be written with the $3\frac{1}{2}$ yds. expressed as 3 yds. 1 ft. 6 in.

Sq. rds. *Sq. yds.* *Sq. ft.*

12 6 7

$30\frac{1}{4}$

366

3

369 yds.

9

6 m 0)3328 ft.

$55\frac{28}{60}=\frac{7}{15}$

Ex. A garden is 12 rds. 6 yds. 7 ft. in area. One side is 60 ft. in length; find the length of the other.

Since the area is found by multiplying the length and breadth together, we shall find the breadth to be $55\frac{7}{15}$ ft., by dividing the area 3328 sq. ft. by the length, which is 60 ft.

EXERCISE 51.

☞ The first 11 examples to be solved mentally.

1. What part of a foot in length is 8 in.? 6 in.? 4 in.? 3 in.? 2 in.?

2. What part of a yard of rope is 18 in.? 2 ft.? 12 in.? 9 in.? 4 in.? 3 in.? 2 in.?

3. What part of a square foot is 72 in.? 48 in.? 36 in.? 24 in.? 12 in.? 6 in.? 3 in.?

4. What part of a sq. yard of sheet-iron is 3 ft.? 6 ft.? 1 ft.? 2 ft. 36 in.? 4 ft. 72 in.? 6 ft. 108 in.?

5. An acre of farm land is worth $320; what must I pay for 80 rods? 40 rods? 32 rods? 20 rods? 16 rods?

6. If a wire fence cost 30c. per yard, what is the price of a foot? and what will be the cost of 2 rods? 1 furlong? 1 mile?

7. A cubic foot of metal is worth $1728; what is the value of 1 solid inch? the ninth part of a cubic yard?

8. A hall-way is 72 inches wide, and takes sixty square yards of matting to cover it; what is its length?

9. A room is 20 yards long and 12 yards wide; how many yards in length of carpet 1 yard wide will be required for it? Find its cost at 75c. a yard.

10. What is the area of a room $3\frac{1}{2}$ yards by $5\frac{1}{7}$ yards? $4\frac{2}{3}$ yards by $7\frac{1}{2}$ yards?

11. Which is the cheaper of the following prices?

(*a*) 5c. an inch, or 50c. a foot, for lead pipe?
(*b*) 10c. an inch, or $4 a yard, for cloth?
(*c*) $6 a yard, or 50c. an inch, for lace?
(*d*) $1 a yard, or $6 a rod, for fencing?
(*e*) 60c. a sq. foot, or $6.50 a sq. yd., of carpet?
(*f*) 24c. for 8 in., or $1 a yard, of ribbon?

12. Write out tables for Long and Square Measures, as in Art. 201.

13. Express 213 inches in length as yards, etc.

14. How many yards, etc., in 1649 inches of wire?

15. How many inches in 6 rds. 4 yds. 2 ft. 9 in. of distance?

16. What part of a mile is 2 fur. 36 rds. 2 yds.?

17. How many cords of wood are there in a pile 40 yds. long, 2 yds. high and 4 ft. wide?

18. How many fathoms are there in a depth of $\frac{7}{8}$ of a mile?

19. Find the cost of digging a drain 20 feet long. 2 yds. wide, and 14 feet deep, at 90 cents per cubic yard.

20. Find the sum of 12 yds. 4 rds. 3 ft. 25 in.; 33 yds. 2 ft. 10 in.; 7 ft. 132 in.; 10 yds. 7 ft.; 31 yds. 100 in.; 76 yds. 89 in.

☞ In dividing by $30\frac{1}{4}$, bring both divisor and dividend to fourths.

21. The area of a board is 21 feet; its length is 18 in., what is its breadth?

22. What is the area of the four walls of a room, 13 ft. 9 in. wide, 16 ft. 3 in. long, and 14 feet high? and what is the cost of papering it at 2c. a sq. ft.?

23. How many suits of clothes can be made from 13 pieces of cloth, each containing $21\frac{1}{2}$ yds., if it takes $7\frac{1}{6}$ yards to make a suit?

24. In 987,654,321 inches how many miles, etc.?

25. If I have to measure a distance of 3 furlongs with a line three rods and a half long; how many times will the line measure the distance?

26. In 169,500,087,647 sq. inches, how many square miles, etc.?

27. If a plank be $6\frac{3}{4}$ inches wide, what length of it will give a surface of 2 square feet?

28. A block of marble contains 1296 cu. feet, its breadth and depth are each 9 feet; what will be its length?

29. If 17 men reap 19 ac. 97 sq. rods in a day, and 8 of them reap one-third of an acre each; how much ought each of the others to reap?

30. How many sq. yards of block pavement in a street $\frac{1}{2}$ mile long and 1 chain wide.

31. By how much does a road 18 miles long exceed one of 12 mi. 3 fur. 17 rods?

32. I have five farms; the first contains 29 ac. 133 sq. rods, the second 100 ac. 81 sq. rods, the third 85 ac. 69 sq. rods, the fourth 71 ac. 137 sq. rods, and the fifth 112 ac. 98 sq. rods; how much land do I own?

33. If a mountain be $4\frac{7}{8}$ miles high, express its altitude as a fraction of the earth's diameter, which is 7926 miles.

34. What length is that of which 25 yards is $\frac{15}{27}$? of which 7 feet is $\frac{21}{100}$?

35. What length in feet would require to be added to a telegraph wire, which reaches only $\frac{7}{24}$ of the distance between Toronto and Thornhill to complete the whole distance, 12 miles?

36. What part of 2 miles would just measure $\frac{3}{4}$ of 10 rods?

37. A block of land is $1\frac{1}{4}$ miles square, how many acres would there be in $\frac{1}{10}$ of it?

38. Divide $10\frac{1}{4}$ square miles of land among 33 persons and give each an equal share; what would each receive?

---

220. Time is measured by seconds, minutes, hours, days, weeks, months and years, as shown by the following

TABLE OF TIME.

60 seconds (sec.)	make	1 minute, min.
60 minutes	"	1 hour, hr.
24 hours	"	1 day, da.
7 days	"	1 week, wk.
365 days	"	1 year, yr.

221. The months of the year are January, February, March, April, May, June, July, August, September, October, November and December.

April, June, September and November have each 30 days, and all the others have 31 days, except February, which has 28 and sometimes 29 days.

☞ The year in which February has 29 days is called **Leap Year**, and thus has 366 days.

Leap Years are exactly divisible by 4, such as 1860, 1880, 1888.

Ex. How many hours from 12 o'clock, noon, on the 15th of December, to 12 o'clock, midnight, on the 10th of January next?

Since December has 31 days, there will be 16 days up to noon, and 12 hours more on the last day of December up to midnight. From this time to January 10th gives 10 days more; in all 16+10, or 26 days and 12 hours.

26 days = 26 × 24 hours = 624 hours.
624 + 12 = 636 hours.

EXERCISE. 52.

☞ The first seven questions to be solved mentally.

1. How many minutes in $\frac{1}{2}$ an hour? in $\frac{3}{4}$ of an hour? in $\frac{1}{4}$ of an hour? in $1\frac{1}{2}$ hours? in 4 hours?

2. What part of an hour is 30 minutes? 45 minutes? 20 minutes? 10 minutes? 15 minutes?

3. How many days in 2 wks? in $3\frac{1}{7}$ wks? in the two first months of the year? in 2 years? in $\frac{1}{6}$ of a leap year?

4. What part of a week is 3 days? 24 hrs.? 12 hrs.? $3\frac{1}{2}$ days? 48 hrs.?

5. A man can earn $3 an hour, how much can he earn in a day? in 10 days? in 15 min.? in 1 min.?

6. How long from 6 o'clock in the morning to 8 o'clock at night? from midnight to mid-day?

7. Which is the cheaper of the following rates:

   (*a*) 10c. an hour, or $2 a day, for a cab?
   (*b*) $2 a day or $14.50 a week?
   (*c*) 1c. a min. or 50c. an hour?
   (*d*) 10c. for 20 min. or 40c. an hour?
   (*e*) $24 for 3 mos. or $100 a year?
   (*f*) $2 a day or $62 for the month of April?

8. How many seconds in a year? in a leap year? in the month of June? in the month of August? in the month of February, 1880? in a week? in a day? in an hour?

9. How many minutes in the year 1882? in the year 1884? in the month of July? in September? in the longest month in the year? in the shortest month of

the year? in the first three months of 1888? in the last three months of the year?

10. How many seconds in 5 hrs. 15 min. 12 sec.?

11. How many hours, etc., in 38497 sec.?

12. How many days from April 1st to Oct. 19th?

13. Reduce 27789 min. to weeks, etc.

14. What part of a week is 6 days, 12 hrs. and 30 min.?

15. Find the difference between $\frac{9}{16}$ of a day and $\frac{4}{5}$ of week.

16. How many days from August 24th, 1883, to August 12th, 1884?

17. How many minutes from 18 min. past 9 to 32 min. to 10 in the morning?

18. What part of a week is wasted by a boy who is idle for 2 days 10 hours?

19. Find the third part of 3 wks. 6 da. 14 hrs. 17 min. 57 sec.

20. What part of a day is 1 hr. 52 min. 30 sec.?

21. What must be taken from $\frac{2}{5}$ of 14 da. 9 hrs. to leave half an hour and 8 min.?

22. Find the value of $\frac{9}{10}$ of a day $+\frac{7}{9}$ of 1 hr. $-\frac{1}{3}$ of 13 hrs. 2 min. 6 sec.

23. From half-past 5 p.m., on the 30th of June, to 20 min. to 11 a.m., on the 5th of September, how much time elapses?

24. The true year contains 365 days, 5 hrs., 48 min., $49\frac{7}{10}$ sec.; how many days, etc., in 12 years?

25. Express $\frac{8}{13}$ of an hour in terms of a minute, of a day, and of a week.

26. What is the period of which 3 hrs. 20 min. is $\frac{5}{9}$? of which three days is $\frac{6}{7}$?

27. Of what time is 3 min. 10 sec. the seven-fifths?

28. What time would you have to add to 3 hrs. 5 min. 20 sec. so that it would become $\frac{5}{6}$ of a day?

29. How long would it take a man to complete a journey of 20 miles at the rate of $3\frac{1}{2}$ miles an hour? And if he started at 10 a.m., at what time by the clock would he finish it?

30. Two men start to walk, the one from Toronto, the other from Newmarket, a distance of 32 miles, at 9 a.m., the one walks at the rate of 3 miles an hour, the other $3\frac{1}{2}$ miles an hour; at what time by the clock will they meet?

31. A boy studies on Monday 5 hrs. 30 sec., $\frac{3}{4}$ of $\frac{11}{12}$ of a day on Tuesday, $\frac{5}{7}$ of 12 hours on Wednesday, $\frac{2}{3}$ of 10 hrs. 20 min. on Thursday, and $\frac{5}{6}$ of 8 hrs. 10 min. 40 sec. on Friday; how many hours would he study during a school week?

32. A girl commenced to read a book of 320 pages on Monday at 9 a. m.; if she read every day for 4 hours at the rate of 5 pages in 20 min.; at what time would she finish it?

---

222. Goods are bought and sold, not only by weight, but are often measured: as, a pint of beans, a gallon of milk, a bushel of potatoes, a hogshead of wine.

223. Liquids, such as wine, ale, etc., are measured in a different way from fruits, grain, etc.

224. The latter, and such commodities as are taken up in the hand, are measured by the following

TABLE OF DRY MEASURE.

2 pints (pt.)	make	1 quart, qt.
4 quarts	"	1 gallon, gal.
2 gallons	"	1 peck, pk.
4 pecks	"	1 bushel, bush.

☞ The standard measure in Canada is the "Imperial Gallon," containing $277\frac{274}{1000}$ cubic inches.

The law determines the weight of a bushel of different kinds of produce, as follows:

Oats,	34 lbs.	Corn,	56 lbs.
Barley,	48 lbs.	Wheat,	60 lbs.
Buckwheat,	48 lbs.	Beans,	60 lbs.
Timothy Seed,	48 lbs.	Peas,	60 lbs.
Flax Seed,	50 lbs.	Clover Seed,	60 lbs.
Rye,	56 lbs.	Potatoes,	60 lbs.

225. Wine, ale, etc., are measured by the following

TABLE OF LIQUID MEASURE.

4 gills (gi.) make 1 pint, pt.
2 pints " 1 quart, qt.
4 quarts " 1 gallon, gal.

☞ A hogshead (hhd.) of wine contains 63 gals.
A hogshead of beer or ale " 54 gals.
A barrel of beer or ale " 36 gals.

EXERCISE 53.

☞ The first 8 questions to be solved mentally.

1. In a quart of water how many gills? how many half pints?

2. If hickory nuts are 20c. a quart, what is the price of a pint? a half-pint? a peck? a quarter of a peck? a half-bushel? three-quarters of a bushel?

3 I paid 80c. for a gallon of Fulton & Michie's best vinegar: how much was that a quart? a half-pint? for 6 gills? for half a gallon?

4. Bought a bushel of Spanish chestnuts from Cleghorn & Son for $4; what was the price for half a gallon? for a quart? for 3 bushels? for 4 quarts?

5. Bought from Farmer Jones, potatoes at 60c. a bushel; in order to gain 10c. on every peck I sell, what must I charge for a bushel? for $\frac{1}{5}$ of a peck? for a bushel and a quarter? for half a peck?

6. What part of a bushel of corn is 3 pks? $\frac{1}{2}$ a gallon? 4 gals.? 8 qts.? 8 pints?

7. Which is the cheaper of the following prices:

(*a*) $1 a bushel, or 25c. a peck?
(*b*) 20c. a qt., or 10c. a pint?
(*c*) 5c. a gallon, or 60c. a bushel?
(*d*) $2 a quart, or 30c. a gill?
(*e*) 80c. a peck, or 12c. a gallon?

8. At 30c. a gallon, what is the cost of a hogshead of ale? a quart of nuts?

9. How many dozen of pint bottles could be filled from 20 barrels of ale?

10. By how much does the number of gals. in 10000 pts. of wine exceed the number of hogsheads in 10000 gallons?

11. How many pints of water in 6 gals. 3 qts. 1 pt.?

12. How many bushels, etc., of nuts in 1387 qts.?

13. How many hogsheads of wine in 6324 gills?

14. I bought 4 barrels of spirits, each holding $15\frac{3}{4}$ gallons, at $3\frac{1}{2}$c. a gill; what can I sell it for, so as to gain $20?

15. How many gallons of cider at $12\frac{1}{2}$c. a gallon can I obtain in exchange for 130 bushels of apples, at 35c. a bushel?

16. What is the value of 325 bags of beans, each containing 2 bush. 1 pk., at 70c. a bushel?

17. How many loads of apples each containing $27\frac{3}{4}$ bush. at 45c. a bush., can be bought for $49.95.

18. What part of 1 gallon is 2 qts. $1\frac{1}{4}$ pt.?

19. If 376 gals. 3 qts. 1 pt. of milk be divided equally among 9 charities, how much will each receive?

20. A man bought $\frac{57}{64}$ of a bushel of nuts and sold them at 10c. a quart, but only gave $1\frac{1}{2}$ pts. to a quart; how much did he receive?

21. A man bought 14 bags of beans, each holding 2 bush. 2 pks., for $21, and sold them in boxes of 1 bush. 3 pks. each; find the price per box.

22. What part of 62 gals. 2 qts. 1 pt. must be taken from a hogshead of wine to leave 42 gals. 1 pt.?

23. If a vessel is $\frac{3}{5}$ full, and after 70 pints are drawn off, is found to be $\frac{3}{8}$ full, how much did it contain?

24. If a barrel of ale is $\frac{9}{10}$ empty, how many quarts are still left in it?

25. How many barrels of ale could be filled from a vat containing 1296 gallons? and how many dozen quart bottles would this quantity fill?

26. How many bushels of clover seed in 5 loads, each weighing 1 ton 36 lbs.?

27. How many bushels of oats would weigh as much as 560 bushels of corn?

28. How many bushels of oats would a horse eat in 1 year, if fed three times a day, and given 5 quarts at each meal?

29. What part of a bushel of potatoes would weigh as much as $\frac{2}{3}$ of a bushel of buckwheat?

30. How many barrels of flour of 196 lbs. could be made from 1000 bushels of wheat, if it takes 3 lbs. of wheat to make 2 lbs. of flour?

---

226. Many articles of merchandise are bought and sold by special names, the most important of which compose the following

GENERAL TABLE.

1 dozen (doz.)	= 12 articles.
1 score	= 20 "
1 quire	= 24 sheets.
1 ream	= 20 quires.
1 gross	= 12 dozen.
1 great gross	= 12 gross.
1 stone	= 14 pounds.
1 brl. of flour	= 196 "
1 " pork or beef	= 200 "

EXERCISE 54.

☞ The following questions should be solved **mentally:**

1. How many articles in 3 dozen? in $5\frac{1}{2}$ dozen? in 2 score? in 1 score and 3 dozen? in 3 score and ten? in $6\frac{3}{4}$ dozen? in one-half a score? in $\frac{1}{3}$ of a dozen $+\frac{1}{5}$ of a score? in 5 more than $1\frac{1}{4}$ dozen?

2. At 60c. a dozen, what is the cost of half a dozen oranges? of 2 score apples? of 5 peaches? of half a score of lemons? of a gross of pens at 5c. a doz.?

3. What part of a score is a dozen?

4. If foolscap paper costs 1c. a sheet, what must I pay for a quire? for $2\frac{1}{2}$ dozen sheets? for a ream? for 8 sheets more than half a quire? for 2 quires less 11 sheets? for $\frac{1}{4}$ of a ream?

5. Which is the cheaper of the following prices:—

   12c. a dozen, or 20c. a score?
   3c. a piece, or 30c. a dozen?
   50c. a quire, or 2c. a sheet?
   $\frac{1}{2}$ a dozen for 30c., or 80c. a score?
   $1\frac{1}{2}$c. each, or 20c. a dozen?
   2 score for a shilling, or $\frac{1}{2}$ shilling a score?

6. A bought eggs for 30c. a dozen, how much per dozen must he sell them for to gain 1c. apiece? to gain 15c. a dozen? to lose $\frac{1}{2}$c. each? to gain 20c. a score? to lose 10c. a score?

7. How many score of sheep in 5 flocks, each containing 9800 sheep?

8. How many boxes, each containing 1 gross, are there in nine million pens?

9. What would 120 gross of spools cost at 50c. a dozen?

10. How many barrels in 1120 stones of flour; and what would the whole cost at $3 per cwt.?

11. From 60 reams of paper, how many books of 400 pages each could be made, if one sheet is folded into 8 leaves?

12. How many stones of flour will weigh as much as 21 barrels of pork?

13. How many barrels of beef can be packed from 20 carcases, each weighing 1150 pounds, and what would the whole cost at $16.50 per barrel?

14. What part of a stone would weigh as much as $\frac{1}{28}$ of a barrel of a flour?

15. If 10 eggs weigh a pound, how many dozen would weigh as much as a hog, that would make $2\frac{1}{2}$ barrels of pork?

16. Three-fourths of a ream is what part of 10 quires? of a book containing 800 pages? of 450 sheets?

17. A grocer in packing 6 dozen dozen eggs, broke half a dozen dozen, and sold the remainder at 20 cents a dozen. How much did he receive for the eggs?

18. How many reams of paper will print an edition of 2500 copies of the Royal Canadian Reader No. 4, each copy containing 384 pages, if one sheet make 12 leaves?

19. What is the cost of a great gross of steel pens, at $6\frac{1}{4}$ cents for $\frac{3}{4}$ of a dozen?

20. At $22.86 per ream, what will 9 reams, 6 quires, 12 sheets of paper cost?

21. How large an edition of an Elementary Arithmetic can be printed from 99 reams of paper, allowing 16 sheets to the volume?

22. How many dozen cabbages can be planted 2 feet apart, in a garden containing 132 rows two feet wide, and 20 rods long, and what would they be worth, when brought to market at 40 cents per dozen? How many acres are there in the garden?

CHAPTER V.

DECIMALS.

227. In the previous system of notation, we have seen that the figures *increase* ten fold from right to left. That is, *decrease* tenfold from left to right. This can be carried out in the same manner to the right of the units' place.

228. In the number 843·5·267, if we call the 5 units, the values to its left increase, and those to its right decrease tenfold for each place.

Thus, 5 means 5 units.
3 means 3 tens of units.
2 means 2 tenths of a unit.
4 means 4 hundreds of units.
6 means 6 hundredths of a unit.
8 means 8 thousands of units.
7 means 7 thousandths of a unit.

And so on for any number.

229. The part 267 is called a decimal, and we shall see that it may be expressed as a fraction, for

$$\cdot 267 = \tfrac{2}{10} + \tfrac{6}{100} + \tfrac{7}{1000} = \frac{200+60+7}{1000} = \frac{267}{1000}$$

230. The expression 8435·267 will be read :—8 thousand 4 hundred and thirty-five, *decimal two six seven*, (not decimal two hundred and sixty-seven.)

It may also be read :—8 thousand, four hundred and thirty-five, *and two hundred and sixty-seven thousandths.*

☞ The pupil should be well exercised in writing decimals rapidly, when they are read in the latter way.

231. The dot (·) on the right of the units' figure in Art. 228 is called the Decimal Point, and marks the beginning of the decimal part, and the dot on the left is not written in practice.

232. *Hence any decimal can be written in a fractional form by using the figures of the decimal for the numerator, and writing for the denominator, 1, followed by as many ciphers as there are places in the decimal.*

Ex. 1. $\cdot02605 = \frac{2605}{100000}$
Ex. 2. $12\cdot006 = 12\frac{6}{1000}$

233. *For the same reason, we can express the fractional form in a decimal form, by using the numerator for the decimal part and counting off from the right as many places as there are ciphers in the denominator, supplying ciphers to the left, if necessary.*

Ex. 3. $-\frac{502}{10000} = \cdot0502.$
Ex. 4. $-\frac{7503}{100} = 75\cdot03.$

234. All examples given in the decimal form may, therefore, be worked by Vulgar Fractions, but Addition and Subtraction may also be performed just like whole numbers, care being taken, as before, that only like orders are added together, or subtracted from one another.

Ex. 5. Find the sum of 4·02, ·0075, 16·31, and 41·032.

$4\cdot02 = 4\frac{2}{100}$; $\cdot0075 = \frac{75}{10000}$
$16\cdot31 = 16\frac{31}{100}$; $41\cdot032 = 41\frac{32}{1000}$
$4+16+41=61$
$\frac{2}{100}+\frac{75}{10000}+\frac{31}{100}+\frac{32}{1000} =$
$\frac{200+75+3100+320}{10000} = \frac{3695}{10000} = \cdot3695.$

Hence the sum is 61·3695.

This may also be worked thus :—

$$\begin{array}{r} 4\cdot02 \\ \cdot0075 \\ 16\cdot31 \\ 41\cdot032 \\ \hline 61\cdot3695 \end{array}$$

Ex. 6.—Find the difference between $21\frac{7}{1000}$ and 40·935.

$$21\tfrac{7}{1000} = 21{\cdot}007$$

$$\begin{array}{r} 40{\cdot}935 \\ 21{\cdot}007 \\ \hline 19{\cdot}928 \end{array}$$

This may also be worked thus :—

$$40{\cdot}935 - 21\tfrac{7}{1000} = 40\tfrac{935}{1000} - 21\tfrac{7}{1000} = 19\tfrac{928}{1000}.$$

Ex. 7.—Multiply ·037 by ·042.

$$\cdot 037 = \tfrac{37}{1000}\ ;\ \cdot 042 = \tfrac{42}{1000}$$

$$\tfrac{37}{1000} \times \tfrac{42}{1000} = \tfrac{1554}{1000000} = \cdot 001554.$$

This may also be worked thus :—

$$\begin{array}{r} \cdot 037 \\ \cdot 042 \\ \hline 74 \\ 148 \\ \hline \cdot 001554 \end{array}$$

235. *From this we see that we multiply as if they were whole numbers, and then mark off from the right of the product as many decimal places as there are in the multiplier and multiplicand together, supplying ciphers to the left, when necessary.*

Ex. 8.—Multiply 31·25 by ·0196.

$$\begin{array}{r} 31{\cdot}25 \\ \cdot 0196 \\ \hline 18750 \\ 28125 \\ 3125 \\ \hline \cdot 612500 \end{array}$$

☞ In this result the two ciphers on the right of the decimal have no value, and may be struck off, leaving ·6125.

Ex. 9.—Divide ·0296 by ·08.

$$\cdot 0296 = \tfrac{296}{10000}\ ;\ \cdot 08 = \tfrac{8}{100}.$$

$$\tfrac{296}{10000} \div \tfrac{8}{100} = \tfrac{296}{10000} \times \tfrac{100}{8} = \tfrac{37}{100} = \cdot 37.$$

☞ Proof: $\cdot 08 \times \cdot 37 = \tfrac{8}{100} \times \tfrac{37}{100} = \tfrac{296}{10000} = \cdot 0296.$

This Example may also be worked thus:

$$\begin{array}{r} \cdot 08)\cdot 0296(\cdot 37 \\ \underline{24} \\ 56 \\ 56 \end{array}$$

Now, since there are *four* places of decimals in the dividend, *two* in the divisor and *two* in the quotient, we see that the number of places in the quotient can always be found by taking the number of places in the divisor from the number of places in the dividend and marking off this difference as the number of places in the quotient.

We also notice that 37 is obtained by dividing 8 into 296, just as if they were whole numbers.

236. From this we obtain the following

RULE FOR DIVISION OF DECIMALS.

Divide the numbers as if they were whole numbers, and mark off from the right of the quotient, the number of places there are in the dividend more than in the divisor, prefixing ciphers to the left, if necessary.

☞ Since a decimal is not affected in value by adding ciphers to the right, we can thus always make the dividend have more places than the divisor.

Ex. 10.—Divide 52·3 by ·125.

The dividend may be written 52·3000.

$$\begin{array}{r} 125)523000(4184 \\ \underline{500} \\ 230 \\ 125 \\ \hline 1050 \\ 1000 \\ \hline 500 \\ 500 \end{array}$$

In the divisor, ·125, we have 3 places; in the dividend 52·3000, we have 4 places; hence in the quotient we must have 1 place, which gives 418·4.

☞ PROOF :— $418\cdot4 \times \cdot125 = 52\cdot3000 = 52\cdot3$.

237. In many cases the division will be very long, and very often will not terminate, and it is then required to find the quotient to a certain number of places, thus :—

Ex. 11.—Find the quotient to *four places* of *decimals* when 2·51 is divided by ·109.

```
109)25100000(230275
   218
   330
   327
     300
     218
      820
      763
       570
       545
```

Now, there are 3 places of decimals in the divisor, and since we are to have 4 places of decimals in the quotient, there must be 7 places in the dividend, or 2·5100000, and the operation will be performed as shown, and the quotient will be 23·0275.

EXERCISE 55.

☞ The pupil should prove the results in each question.

(*a.*) Express as ordinary fractions in their simplest form:—

1. ·495; ·0075; 12·8; 68·1875.
2. ·375; ·225; ·0068; ·3125.
3. ·95; ·875; ·4375; 8·275.

(*b.*) 1. The distance from A to B is 42·3 chains, from B to C is 13·06 ch., from C to D is 8·049 ch., from C to D is 1·6 ch., from D to E is ·037 ch. How far from A to E?

2. A man had 1000 acres of land, and sold 450·625 acres; how many acres has he left?

3. From ·00038 take 36 ten-millionths.

4. What must be taken from 34·634 acres to leave 28·9483?

5. To how many pounds of chicory must 23·859 pounds of coffee be added to produce a mixture of 29·796 pounds?

6. What is the difference between 100 miles and ·03846 miles?

7. What is the sum of 18 thousandths, 15 millionths, 81 hundredths, 146 ten-thousandths, 834 hundred-thousandths?

8. What is the cost of 34·5 yds of cloth, at $3.15 per yard?

9. Since 16·5 feet make a rod, how many feet are there in 23·7 rods?

10. The product of two numbers is ·0048; one is ·06; what is the other?

11. Divide ·04905 by ·327.
135·05 by ·037.
71·142 by ·0071.

12. Divide each to four places of decimals:—

150·75 by 30·25.
300·402 by 12·1.
4·00334 by 6·31.

13. Find the quotients each to five places of decimals.

·3412 ÷ 8·4736.
·004134 ÷ ·3243.
·00079085 ÷ ·83497.

---

238. We have seen that any decimal may be expressed as a vulgar fraction; we will now reduce a vulgar fraction to its equivalent decimal.

The fraction $\frac{3}{5}$ is equal to $\frac{6}{10}$, which may be written ·6. Again, $\frac{1}{25}=\frac{4}{100}=$·04.

The former might have been obtained by dividing 3·0 by 6, thus:—

$$6\overline{)3\cdot0}\quad\cdot5$$

and the latter by dividing 1·00 by 25, thus:—

$$25\overline{)1.00}\quad\cdot04$$

239. Hence we see that, while such fractions as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., may be known by inspection to be ·5, ·25, ·2, etc., any fraction may be reduced to a decimal by simply dividing the numerator by the denominator.

Ex. 1.—Express $\frac{5}{16}$ as a decimal.

16)5·0000(·3125
48
20
16
40
32
80
80

Here, the value of the numerator is not altered by adding ciphers to the right of the decimal point.

It will then be seen that the position of the decimal point is found as in ordinary division of decimals.

Ex. 2.—What decimal of a pound is the same as $\frac{3}{32}$ of a pound?

4)3·0000
8)·7500
·09375

Here the division is carried on by means of the factors of 32, that is: 4 and 8. Therefore, $\frac{3}{32}$ of a pound is the same as ·09375 of a pound.

The previous results may be proved thus;—

$$\cdot 3125 = \tfrac{3125}{10000} = \tfrac{5}{16}.$$
$$\cdot 09375 = \tfrac{9375}{100000} = \tfrac{3}{32}.$$

240. We have seen in Art. 237 that, in many cases, the division will not terminate, in other words, there will be a remainder, no matter how far we proceed.

Ex. 3.—Reduce $\frac{28}{45}$ to a decimal.

45)28·000)·622
270
100
90
100
90
10

Here the remainder, 10, is continually repeated; then we see that the figure 2 in the quotient must also be repeated, the quotient being written $\cdot\dot{6}\dot{2}$.

Ex. 4.—Express $\frac{103}{165}$ as a decimal.

165)103·000(·$\dot{6}\dot{2}\dot{4}$
990
400
330
700
660
400

The remainder, 40, repeats, and therefore the figures in the quotient from the 2 will repeat, and the quotient is written in the form $.\dot{6}\dot{2}\dot{4}$.

241. Such expressions are called Circulating **Decimals, or** Repeating Decimals, from the fact that one or more figures in them circulate or repeat.

☞ The result of Ex. 1 would, if carried out, be written ·62222, etc., and in Ex. 2 the quotient would be written ·6242424, etc.

The dots are placed over the figure or figures that repeat. Thus:

$\dot{0}5\dot{7}2$ means ·057205720572, etc.

$62\dot{9}4\dot{7}$ means ·62947947947, etc.

242. Decimals like $\dot{0}5\dot{7}2$, where all the figures after **the** point repeat. are called **Pure Circulating Decimals.**

Decimals, like $62\dot{9}4\dot{7}$, where some of the figures after the point do not repeat, are called **Mixed Circulating Decimals.**

243. These decimals are reduced to vulgar fractions by the following

RULE.

If the decimal is a Pure Circulating Decimal, place the repeating part of the decimal for the numerator, and for the denominator as many nines as there are figures in the repeating part.

If a Mixed Circulating Decimal, subtract the part that does not repeat from the whole decimal, place the difference for the numerator, and for the denominator as many nines as there are figures in the repeating part, followed by as many ciphers as there are figures that do not repeat in the decimal.

Ex. 1.—$\cdot\dot{0}3\dot{7} = \frac{37}{999}$.

Ex. 2.—$\cdot 03\dot{7} = \frac{34}{900}$.

Subtracting 3 from 37 leaves 34, which is placed as numerator.

Ex. 3.—$5\cdot\dot{8}9\dot{2}$.

$5892 - 58 = 5834$.

Thus, the required fraction will be

$\frac{5834}{990}$ or $5\frac{884}{990}$.

☞ These results may be proved by dividing the numerator by the denominator

244. All examples on Circulating Decimals are worked by reducing the decimals to fractions. The operations are then performed as in fractions, and the result thus obtained expressed as a decimal.

Ex. 1.—$2{\cdot}7\dot{4}0\dot{2}\dot{4}\dot{3} \times {\cdot}\dot{2}\dot{4}\dot{3} = \frac{27385}{9999} \times \frac{243}{999} = \frac{2}{3} = {\cdot}\dot{6}$.

Ex. 2.—$\cdot\dot{2}\dot{2} \div \cdot\dot{1}\dot{3} = \frac{22}{99} \times \frac{99}{12} = \frac{10}{6} = 1{\cdot}\dot{6}$.

EXERCISE 56.

(*a*) Express the following fractions as decimals:—

1. $\frac{5}{8}$. 2. $\frac{1}{16}$. 3. $\frac{17}{20}$. 4. $\frac{13}{25}$. 5. $\frac{14}{40}$. 6. $\frac{61}{24}$.
7. $\frac{3}{128}$. 8. $\frac{5}{128}$. 9. $24\frac{3}{5}$. 10. $\frac{21}{64}$. 11. $\frac{3682}{31125}$. 12. $\frac{9386}{6250}$.
13. $\frac{1}{37}$. 14. $\frac{1}{13}$. 15. $\frac{41}{333}$. 16. $\frac{5}{21}$. 17. $\frac{103}{105}$. 18. $\frac{158}{495}$.

(*b*) Express the following decimals as fractions in their lowest terms:—

1. $\cdot\dot{7}20\dot{5}$. 2. $\cdot\dot{7}\dot{2}$. 3. $\cdot\dot{1}3\dot{5}$. 4. $\cdot\dot{0}9\dot{3}$. 5. $\cdot2356\dot{3}$.
6. $\cdot0169$. 7. $\cdot20\dot{4}\dot{5}$. 8. $\cdot10\dot{4}\dot{5}$. 9. $\cdot02\dot{4}39$. 10. $\cdot00006$.

(*c*) 1. Multiply $55{\cdot}6\dot{9}$ by $\cdot\dot{3}$.

2. Find the product of $5{\cdot}\dot{4}1463$ and $\cdot\dot{1}2\dot{3}$.

3. Divide $\cdot0\dot{8}\dot{2}$ by $\cdot\dot{6}$.

4. Find the quotient of $31{\cdot}7\dot{9}\dot{1}$ divided by $3{\cdot}9\dot{7}\dot{3}$.

5. Divide $\cdot2\dot{9}\dot{7}$ into $2{\cdot}2\dot{9}\dot{7}$.

6. The product of two decimals is $1\frac{4}{5}$; one of them is $\cdot2\dot{4}\dot{5}$, find the other.

CHAPTER VI.

PERCENTAGE.

245. The word **Percentage** occurs so often in ordinary business affairs, that everyone should know its meaning.

If a man has 100 gallons of vinegar, and loses 20 gallons, he is said to have lost 20 per cent. If a man has 100 dollars, and by trading, increases it to 140 dollars, he is said to have gained 40 per cent.

246. The term **Per Cent.** thus means per hundred, and is only a short way of writing **per centum** (per hundred.)

☞ We see that 20 per cent. means 20 on every hundred; therefore, 20 per cent of anything, is the same as $\frac{20}{100}$, or $\frac{1}{5}$ of it.

Thus:—80 per cent. of 150 pounds of sugar is the same as $\frac{80}{100}$ of 150, or $\frac{4}{5}$ of 150, which equals 120 pounds.

☞ The expression % stands for **per cent.**; thus, 5% means 5 per cent., or $\frac{1}{20}$.

Ex. 1. What is 8% of 150 pounds?

$$8\% = \frac{8}{100} = \frac{2}{25}.$$

$\frac{2}{25}$ of 150 lbs. = 12 pounds.

Ex. 2. What is 120% of 80 yards?

$$120\% = \frac{120}{100} = \frac{6}{5}.$$

$\frac{6}{5}$ of 80 yards = 96 yards.

Ex. 3. A man lost 8 horses out of a purchase of 400; what did he lose %?

Here we are to find what he lost on 100 horses, if he lost 8 on 400.

400 gave a loss of 8, therefore 100 gave a loss of 2. That is, he lost 2 per cent.

PROOF: $2\% = \frac{2}{100} = \frac{1}{50}$, and $\frac{1}{50}$ of $400 = 8$.

That is 8 horses lost.

Ex. 4. A man had a flock of 800 sheep, and lost 60% of them by disease; how many sheep were left?

Since he lost 60%, he must have (100% − 60%,) or 40% left.

$$40\% = \frac{40}{100} = \frac{2}{5}.$$

$\frac{2}{5}$ of $800 = 320$ sheep.

Compare this with Ex. 5, page 145.

247. The expressions *on* and *off* occur in a peculiar sense in percentage.

A merchant selling his goods (bought in England) at 60% *on* would sell them for 60% or $\frac{3}{5}$ more than their total cost price. Thus 60% *on* cost, means $\frac{3}{5}$ more than cost, or $\frac{8}{5}$ of the cost.

Hence 20% *on* 60c. a yard is $\frac{6}{5}$ of 60c., or 72c. a yard.
40% *on* 10d. a pound is $\frac{7}{5}$ of 10d., or 14d. a pound.

Again, a farmer selling a plough at 20% *off* the cost would just receive 80%, or $\frac{4}{5}$ of the cost.

Hence 15% *off* $40, would leave 85% or $\frac{17}{20}$ of $40, that is $34, making a loss of $6.

30 % *off* 80 bushels would be 70% or $\frac{7}{10}$ of 80 bushels, which is 56 bushels.

EXERCISE 57.

☞ The first 12 questions should be solved mentally.

1. Express the following percentages in their simplest fractional form:—

1%.	2%.	4%.	5%.	10%.	$12\frac{1}{2}$%.	15%.
$17\frac{1}{2}$%.	20%.	25%.	30%.	$33\frac{1}{3}$%.	35%.	40%.
45%.	150%.	90%.	75%.	$66\frac{2}{3}$%.	55%.	180%.

2. What % of a man's income is $\frac{3}{4}$ of it? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{9}{10}$? $\frac{20}{50}$? $\frac{1}{3}$? $\frac{25}{25}$? $\frac{7}{5}$? $\frac{12}{10}$? $\frac{7}{4}$? $\frac{5}{5}$?

3. What is the difference between 30% of my farm and $\frac{2}{5}$ of it? $\frac{8}{10}$ of a lot and 60% of it? 50% of $800 and 40% of $800? $\frac{3}{4}$ of 80 pks. and 80% of 80 pks? 15% of 200 acres and $\frac{1}{4}$ of 200 acres?

4. A man owned 60% of a farm of 640 acres and sold $\frac{1}{2}$ of it; how many acres had he left?

5. A merchant makes $60 on $200; what is his gain %?

6. A had $600; at the end of the year he has $900; what did he gain %?

7. What is gained per cent., if $\frac{3}{4}$ of a farm is sold for what $\frac{7}{8}$ of it cost?

8. What is the gain %, when goods are sold so that $\frac{5}{7}$ of what they all cost is received for half the goods?

9. Sold $\frac{4}{5}$ of a hhd. of molasses for what the whole cost me; what was my gain per cent.?

10. What per cent. is gained by buying oil at 80 cents a gallon, and selling it at 12 cents a pint?

11. A's money is 50% more than B's; how much per cent less is B's than A's?

12. If I sell $\frac{5}{8}$ of an acre of land for what $\frac{3}{8}$ of it cost, what per cent. do I lose?

13. What is 65% *on* $145? 40% *off* $560?

14. What is $12\frac{1}{2}$% *on* $\frac{7}{9}$ of a shilling? 25% of $\frac{12}{19}$ of a gallon? 20% *off* $\frac{15}{16}$ of a yard? 180% *on* $\frac{35}{42}$ of a dollar?

15. What % *on* $9 is $12? $18? $13.50?

16. A merchant gains 25%; what is the gain in a sale of £250 10s. 5d., and what is the cost price?

17. A merchant loses $12\frac{1}{2}$%; what is the loss in a sale of $721.70, and what was the cost price?

18. What per cent. more than $\frac{5}{9}$ is $\frac{5}{6}$?

19. What per cent. less than $\frac{20}{27}$ is $\frac{8}{15}$?

20. One-fifth is what per cent. of three-fourths?

21. Having used my wagon three years, I sold it for $66, which was 40% less than the cost; what was the cost?

22. If 33⅓% of what is received for goods is gain, what is the gain per cent.?

23. Sold ½ of a section of land in Manitoba for what ⅛ of 5 sections cost; what per cent. was gained on the part sold?

24. A man bought ⅜ of a coal mine, and sold ⅘ of his share for $11,700, which was 30% above the cost; what was the cost of the mine?

25. The population of a town of 18000 inhabitants increases 5% every year; what will be the number of inhabitants at the end of 2 years?

26. A man gains 17½ per cent. on tea; if he sold 400 lbs. and gained $42, what did it cost him a pound?

27. A and B each sold 240 acres of land, A gaining 7½ % and B losing 12½%; if A received $960 more than B, what did they pay an acre for the land?

28. A man bought a farm of 160 acres at $32 per acre; he paid $200 for fencing, $150 for repairing buildings, and $18 for improving the grounds; at what price per acre must it be sold to gain 25% on the entire cost?

29. A farmer raised 20% more wool this year than last; the amount raised during the two years was 1320 lbs.; what amount of wool was raised each year?

30. A man dying, left 33⅓% of his property to his wife, 50% of the remainder to his son, 75% of the residue to his daughter, and the balance, $120, to his servant; how much did each receive?

CHAPTER VII.

PRACTICAL PROBLEMS IN MEASUREMENTS.

248. The pupil should now be able to work the more difficult of those problems requiring a knowledge of Long, Square, and Cubic Measures.

249. The following suggestions, together with those given in Arts. 214, 215, 219, will be found useful in working the problems given:

1. When measures are given in different denominations, they should be expressed in the the highest, instead of being reduced to the lowest, thus:

$$5 \text{ yds. } 2 \text{ ft. } 3 \text{ in.} = 5\tfrac{3}{4} \text{ yds.}$$
$$8 \text{ sq. ft. } 72 \text{ sq. in.} = 8\tfrac{1}{2} \text{ sq. ft.}$$
$$5 \text{ cu. yds. } 9 \text{ cu. ft.} = 5\tfrac{1}{3} \text{ cu. yds.}$$

2. Two measures of length when multiplied together give square measure, hence any square measure divided by a long measure must produce a long measure, thus:

$$\text{Since } 8 \text{ ft.} \times 5 \text{ ft.} = 40 \text{ sq. ft.}$$
$$\text{then } 40 \text{ sq. ft.} \div 5 \text{ ft.} = 8 \text{ ft.}$$
$$\text{or } 40 \text{ sq. ft.} \div 8 \text{ ft.} = 5 \text{ ft.}$$

3. Three measures of length when multiplied together, or a square measure multiplied by a long measure give a cubic measure. Hence a cubic measure when divided by a long measure gives a square measure, or a cubic measure divided by a square measure gives a long measure, thus:

$$\text{Since } 12 \text{ in.} \times 5 \text{ in.} \times 3 \text{ in.} = 180 \text{ cu. in.}$$
$$\text{or } 60 \text{ sq. in.} \times 3 \text{ in.} = 180 \text{ cu. in.}$$
$$\text{Then } 180 \text{ cu. in.} \div 3 \text{ in.} = 60 \text{ sq. in.}$$
$$\text{or } 180 \text{ cu. in.} \div 60 \text{ sq. in.} = 3 \text{ in.}$$

4. In papering a room the area of the paper must be the same as that of the four walls, and its length can be found by dividing this area by the width of the paper, or its width can be found by dividing this area by the length of the paper.

5. The area of the four walls can be easily found by multiplying the length round the room by its height.

6. In painting a surface, if the part not requiring paint be $\frac{3}{7}$, the part to be painted must be the remaining $\frac{4}{7}$.

250. A few examples will show these more clearly.

☞ The numbers in brackets refer to the previous suggestions.

Ex. 1.—A room is 14 ft. 11 in. long, 10 ft. 7 in. wide, and 9 ft. 4 in. high: how many square feet are there in the walls?

14 ft. 11 in. + 10 ft. 7 in. = 25 ft. 6 in. = $25\frac{1}{2}$ ft. = length of the two adjacent walls;
$25\frac{1}{2} \times 2 = 51$ ft. = length of the four walls;
51 ft. $\times$ 9 ft. 4 in. = 51 $\times \frac{28}{3}$ = 476 sq. ft. (5).

Ex. 2.—Find the cost of painting the walls in the preceding example at $42\frac{6}{7}$c. a square yard.

$476 \div 9 = \frac{476}{9}$ = sq. yds. in the walls; $42\frac{6}{7}$c. = $\$\frac{3}{7}$;
$\frac{476}{9} \times \$\frac{3}{7} = \$\frac{68}{3} = \$22\frac{2}{3}$.

Ex. 3.—Find the cost of papering a room 40 ft. 7 in. long, 20 ft. 8 in. wide, and $12\frac{1}{7}$ feet high, with paper $\frac{7}{8}$ yd. wide, and 18c. a yard, if the windows, doors, etc., take up $\frac{1}{5}$ of the walls?

40 ft. 7 in. + 20 ft. 8 in. = $61\frac{1}{4}$ ft. (1);
$61\frac{1}{4} \times 2 = \frac{245}{2}$ = length round the room;
$\frac{245}{2} \times 12\frac{1}{7} = \frac{245}{2} \times \frac{85}{7}$ = number of sq. ft. in the walls (5).

Since $\frac{1}{5}$ of the surface does not require papering, we take $\frac{4}{5}$ of it (6);

Then $\frac{245}{2} \times \frac{85}{7} \times \frac{4}{5}$ = sq. ft. to be papered, and $\frac{245}{2} \times \frac{85}{7} \times \frac{4}{5} \times \frac{1}{9}$ = sq. yds. of paper required.

Now to find the length of the paper we divide its area by its width (4).

Thus $\frac{254}{2} \times \frac{85}{7} \times \frac{4}{5} \times \frac{1}{9} \times \frac{8}{7}$ = length of paper required.

To find its value, multiply its length by the price per yard.

$\frac{245}{2} \times \frac{85}{7} \times \frac{4}{5} \times \frac{1}{9} \times \frac{8}{7} \times 18\text{c.} = \$27.20$.

☞ It will be noticed that the different steps are kept in a fractional form until the end, and then simplified, for by this means we can by cancelling save much labor.

Ex. 4.—A piece of timber with its end 18 inches square contains 5 cu. yds.: find its value when sold at 40c. per foot in length.

18 in. $= \frac{1}{2}$ yd.; $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ yd. = area of the end. Therefore the length must be $5 \div \frac{1}{4} = 20$ yards (3); and 20 yds. = 60 ft., hence 40c. $\times$ 60 = \$24.

EXERCISE 58.

1. Find the length of a table 6 ft. 11 in. wide, and containing 62 sq. ft. 36 sq. in.
2. Find the length of a room 13 ft. broad, the carpeting of which at 4s. 6d. per sq. yard costs £4 19s. 8d.
3. A block of ice is 45 in. long, 52 in. broad, and contains $16\frac{1}{4}$ cubic feet: find its thickness.
4. The papering of a room with paper 30 in. wide at 15c. a yard costs \$10.80: how many square yards in the walls, and if the length be 16 ft. and the breadth 11 ft., find its height.
5. The cost of painting the walls of a room at 12c. a sq. foot is \$110.17: the height is 11 ft. 6 in., and the breadth 15 ft. 8 in., find the length.
6. A path 4 yds. wide around the outside of a garden 85 yds. long and 168 feet broad, is to be paved at 14c. a sq. yd.: find the cost.
7. It takes $136\frac{8}{9}$ yds. of paper for a room 9 yds. 5 in. long, 14 ft. 7 in. broad, and 4 yds. 10 in. high: how many inches wide is the paper?
8. Find the height of a room 20 ft. by 15 ft., the papering of which at 15c. a sq. yd. costs \$14.00.
9. A roof is 50 ft. long by 10 ft. wide, find the cost of covering it with slate 10 in. by $4\frac{4}{5}$ in., if the slates are worth \$6 per thousand.

CHAPTER VIII.

BILLS OR ACCOUNTS.

251. When goods, etc., are sold by one person to another, a **Bill** or **Account** is made out by the seller, and sent with the goods to the buyer, to enable him to see the exact cost of the goods bought.

252. If a firm in England sells a quantity of carpetings to a firm in Toronto, the Bill would be sent as follows:

KIDDERMINSTER, Oct. 20, 1881.

Messrs. JNO. MACDONALD & Co.,
Toronto.

1881. Bo't of EDWARD HUGHES & SONS.

					£	s.	d.
Sept.	5	420 yds.	Aubusson Axminster @	5/6	115	10	0
"	"	600 "	Brussels, 5 Frame C. "	3/6	105	0	0
Sept.	12	800 "	" 4 " B. "	3/3	130	0	0
Oct.	3	220 "	Tapestry, "	2/1	22	18	4
					£373	8	4

253. In practice, many short methods are used for calculating the amount of a Bill, and should be carefully remembered and applied as often as possible by the pupil. For example:

Since $\frac{3}{4}$ of a lb. costs as much as 1 lb. all but $\frac{1}{4}$, therefore $\frac{3}{4}$ of a lb. @ 80c. = 80c. − 20c. = 60c.

Also, $\frac{7}{8}$ of a yd. @ \$1.60 = \$1.60 − 20c. = \$1.40.

Again, since $33\frac{1}{3}$c. = $\frac{1}{3}$ of \$1,
24 yds. @ $33\frac{1}{3}$c. would be $\frac{1}{3}$ of the cost of 24 yds. @ \$1, that is \$8.

Since 80c. = $\frac{4}{5}$ of \$1,
75 lbs. @ 80c. would cost $\frac{4}{5}$ of \$75,
that is \$75 − \$15 = \$60.

☞ The mental exercises in Ex. 50, 51, 52, 53 and 54, will be of great assistance in working this class of problems with rapidity.

EXERCISE 59.

Make out, date, and find the amount of each of the following Bills or Invoices:

1. Rice Lewis & Sons, Toronto, sold to Adam & Co., Hamilton: 47 bales Wire @ $1.99; 220 kegs Nails @ $2.95; 270 boxes Glass @ $1.66⅔; 40 kegs Spikes @ $5.87½; 315 boxes Tin Plates @ $4.20.
2. Virginia Tobacco Company, Toronto, sells Simpson, Stuart & Co., Hamilton: 50 caddies "Twin Navy," 19½ lbs. each, @ 30c. per lb.; 25 butts "Dark Sweet," 52 lbs. each, @ 16½c.; 20 boxes "Dark Sweet," 105 lbs. each, at 16c.
3. T. H. Barraclough, Ingersoll, bought of J. D. King & Co., Toronto: 12 pairs French Kid Boots @ $4.25; 24 pairs Misses' Tie Shoes at $1.75; 24 pairs Boys' Shoes @ $1.75; 12 pairs Men's Knee Boots @ $4.50.
4. J. S. Robertson & Bros., Whitby, buy of Canada Publishing Co., Toronto: 12000 Envelopes @ $2.87½ per M; 375 Pass-Books @ 6c.; 29 doz. Slates @ $2.40; 64 doz. Lead Pencils @ 37½c.; 18 doz. Maps (assorted) @ $27.75.
5. Geo. Lewis sold to Capt. McMaster: 25 lbs. Sugar @ 9c.; 40 lbs. Maple Sugar @ 18¾c.; 6 lbs. Cheese @ 12½c.; 8 lbs. Butter @ 23c.; 4 lbs. Raisins at 15c.; 2 lbs. Cream Tartar @ 45c.
6. Chas. Pollock, Toronto, sells to W. Brown, Barrie: 28¼ yds. Silk @ $2.60 per yd.; 47¾ yds. Lace @ $14 per yd.; 28⅞ yds. Velvet @ $2.80 per yd.; 16½ doz. Silk Spools @ 60c. per doz.
7. P. B. Ball & Co., Toronto, sell to Clinton E. Brush & Bro., Toronto: 500 doz. Knitted Squares @ $9.35 per doz.; 205¾ doz. Kid Gloves @ $11.60 per doz.; 406 doz. Child's Hose @ $2.25 per doz.; 100 Men's Scarfs @ 8¾c. each.

EXAMINATION PAPERS.

The following papers are similar to those given at the examination for entrance to High Schools and Collegiate Institutes.

PAPER I.

Ex. 1.—Divide $80 between A and B, so as to give A $30 more than B.

Suppose A to receive his extra $30 first, then the remainder $50, must be twice the amount each receives after this, that is $25 apiece.

Thus B receives $25.

A " $25+$30=$55.

---

1. Define Greatest Common Divisor, Improper Fraction, Fathom, Per Cent.
2. Express $\frac{7}{11}$ of $\frac{13}{14}$ of $\frac{22}{39}$ as a fraction having 75 for its denominator.
3. A house is worth $576, which is just $156 more than $\frac{3}{4}$ of $2\frac{1}{2}$ times the value of a barn: find the value of the barn.
4. A man sells $10\frac{5}{8}$ lbs. of sugar at the rate of 8 lbs. for $1, and gains 17c.: find the cost of the sugar per lb.
5. What is the greatest number that will exactly divide each of the numbers 1190, 408, 4012, and 1445?
6. I sold a farm for 25 % more than it cost me to A, who sold it to B for $1000, which was $33\frac{1}{3}$ % less than it cost him: what did it cost me?
7. Reduce the fraction $\frac{32177}{36042}$ to its lowest terms.
8. The sixth part of a farm exceeds its eighth part by 20 acres less than its ninth part: how many acres in the farm?
9. Two men had $7583 divided between them; the difference between their shares was $461: what did each get?
10. A room is 67 ft. long, 11 ft. 2 in. high, $38\frac{2}{3}$ ft. wide; the doors and windows take up $\frac{1}{10}$ of the area of the walls and ceiling: find the cost of painting the remainder at 18c. per sq. yd.

PAPER II.

Ex. 2.—If 7 lbs. of coffee are worth as much as 2 lbs. of tea, 3 lbs. of tea worth 11 lbs. of sugar, and $4\frac{1}{2}$ lbs. of coffee cost 48c., what will $19\frac{1}{4}$ lbs. of sugar cost?

1 lb. of coffee $= \frac{48}{4\frac{1}{2}}\text{c.} = \frac{96}{9}\text{c.}$

1 lb. of tea $= \frac{7}{2}$ lbs. coffee $= \frac{7}{2}$ of $\frac{96}{9}$c.

1 lb. of sugar $= \frac{3}{11}$ lbs. tea $= \frac{3}{11}$ of $\frac{7}{2}$ of $\frac{96}{9}$c.

Hence $19\frac{1}{4}$ lbs. of sugar $= 19\frac{1}{4} \times \frac{3}{11} \times \frac{7}{2} \times \frac{96}{9}\text{c.} = \$1.96$.

---

1. Find the smallest number to be taken from 101443929 that the remainder may exactly contain 25203.
2. Divide $213 \times 84 \times 190$ times 264 by 30 times 56 multiplied by 36.
3. Express in Roman numerals 114; 11983.
4. What is the least number of which each of the numbers 144, 240, 480, and 960 is a factor?
5. If $\frac{1}{5}$ of a lb. be worth $66\frac{2}{3}$c., and $\frac{3}{7}$ of a lb. be worth as much as $\frac{1}{14}$ of a bushel, find the value of 100 bushels.
6. If I sell my flour at \$2 a bag I lose \$30, but if I charge \$3 a bag I gain \$30: how many bags were there, and what did they cost me per bag?
7. How much water must I put into a hhd. of wine worth \$336 to make the mixture worth \$1.20 per quart?
8. A and B earn \$840 between them, but $\frac{3}{4}$ of A's equals $\frac{9}{16}$ of B's earnings: what did each earn?
9. If the difference between 75 and $\frac{6}{7}$ of $121\frac{4}{5} + \frac{3}{4}$ of $48\frac{2}{3}$ be taken from $150\frac{1}{2}$, by how many times $\frac{4}{5}$ of $1\frac{1}{8}$ of $4 - 2\frac{1}{4}$ should the result be multiplied to produce $342\frac{63}{100}$?
10. The furniture of a house is worth \$10000, and $\frac{9}{10}$ of $\frac{6}{7}$ of $\frac{5}{9}$ of the furniture costs as much as $\frac{1}{4}$ of $\frac{4}{9}$ of $1\frac{5}{7}$ of the house: find the cost of both together.
11. A farmer gave his eldest son $\frac{11}{141}$ of his farm, and $\frac{8}{65}$ of the remainder to the younger, who thus received 10 acres less than his brother: how many acres in the farm?
12. A and B traded with equal sums of money; A increased his money 20%; B lost \$220, and then had $\frac{1}{2}$ as much as A: what did each start with?

PAPER III.

Ex. 3.—After spending \$60 more than $\frac{5}{8}$ of his money, a man had \$120 left: what had he at first?

If he had spent only $\frac{5}{8}$ of his money he would have had \$60 more left, or \$180.

Then, having spent $\frac{5}{8}$ of his money, he must have $\frac{3}{8}$ of it left.

Therefore, $\frac{3}{8}$ of the money = \$180.
$\frac{1}{8}$ " " = \$60.
$\frac{8}{8}$ " " = \$480.

PROOF: $\frac{5}{8}$ of 480 = 300.
300 + 60 = \$360, amount spent.
\$480 − \$360 = \$120, " left.

---

1. What is a pure Circulating Decimal? How do you reduce it to its equivalent fraction?
   Express as a decimal:
$$\frac{3\frac{1}{3}}{8} + \cdot 625 + \tfrac{1}{8} \text{ of } \frac{7\frac{2}{5}}{\cdot 05} \text{ of } 4 - 1.05.$$
2. One-half the sum of two numbers is 500, and one-half their difference is 300: what are the numbers?
3. If $\frac{1}{5}$ of 56 and also $\frac{3}{7}$ of it be added to itself, the sum is just 26 less than double the number: find the number.
4. How high is a pile of wood 12 yds. long, 12 ft. wide, which contains 18 cords?
5. A can do $\frac{4}{9}$ of a job in $2\frac{2}{3}$ hrs., and B can do $\frac{3}{5}$ of it in $4\frac{1}{2}$ hrs.: how long would they take, working together?
6. I gain 10% by selling vinegar at 22c. a qt.: how many barrels of 63 gals. each did I buy for \$1058.40?
7. If $\frac{1}{3}$ of $1\frac{3}{4}$ be multiplied by $\frac{1}{1\frac{1}{2}}$ of the square of $8\frac{4}{7}$, and $\frac{2\frac{1}{5}}{3\frac{1}{7}}$ be added to the product, from what should the result be taken to produce $\frac{11\frac{3}{32}}{8\frac{7}{8}}$?
8. A man worked $6\frac{3}{4}$ weeks, and, having spent \$15 more than $\frac{4}{9}$ of his earnings, had \$45 left: what did he earn a week?

PAPER IV.

Ex. 4.—A man can row 5 miles an hour in still water: how long will he take to go 21 miles up a stream and back, the stream running 2 miles an hour?

Going up, the stream retards him 2 miles an hour, making his speed only 3 miles an hour.

$21 \div 3 = 7$ hrs., time to go 21 miles up.

Going down, the stream helps him 2 miles an hour, making his speed 7 miles an hour.

$21 \div 7 = 3$ hrs, time to go 21 miles down.

Hence, $7 + 3 \times 10$ hrs., whole time.

---

1. Divide (to four places of decimals) the sum of ·075 and $\frac{75}{10000}$ by the difference between 7·5 and $\frac{75}{100}$.

2. A can do a piece of work in $12\frac{1}{2}$ days of 9 hrs. each, but with B helping him he can do it in $2\frac{2}{5}$ days: how many hours would B alone require to finish it?

3. I gave away $\$35\frac{1}{4}$, and then had \$20.20 more than I gave away: what had I at first?

4. A man paves his garden 49 ft. long and 10 yds. wide at 45c. a sq. yd., except two grass plots each 7 ft. square: what did the paving of it cost?

5. A stream runs half a mile in 15 min., and a man can row a mile in 10 min.: how much longer will he be in going up, than in coming down, a distance of 32 miles?

6. What decimal part of a grain would balance ·000075 lbs. of gold?

7. I paid \$39 to the Queen City Insurance Co. for insurance at $1\frac{1}{5}\%$: what part of my factory, worth \$4550, would I lose in case of fire?

8. Divide 570 acres of land among A, B and C, giving B and C equal shares, but A 60 acres more than B and C together.

PAPER V.

Ex. 5.—A, going 12 miles an hour, gives B, who goes 8 miles an hour, a start of 40 miles: how long before B will be overtaken?

A goes 4 miles an hour more than B, that is, A approaches or overtakes B at the rate of 4 miles an hour, and since he must overtake 40 miles, it will take him $40 \div 4 = 10$ hours.

☞ This applies to the large hand (A) and the small hand (B) of a clock.

A goes 60 min. spaces in 60 min.
B " 5 " " " 60 "

Therefore, A approaches B at the rate of 55 min. spaces in 60 min.

---

1. What must be multiplied by ·08008 to give 48536·488 gals.?
2. From the sum of $\frac{4}{5}$ lb. $4\frac{5}{6}$ oz. and $31\frac{1}{3}$ dwt. take the difference between $\frac{3}{5}$ oz. and $\frac{7}{8}$ dwt.
3. Simplify $\frac{9}{38\frac{1}{4}} \times \frac{174\frac{4}{9}}{196\frac{1}{4}} \times \frac{44\frac{5}{8}}{16\frac{1}{3}} \div \frac{36\frac{4}{7}}{40\frac{8}{11}}$.
4. Two men jointly owned a farm in Manitoba; one had 470 acres more than $\frac{2}{5}$, and the other 790 acres more than $\frac{3}{7}$ of the farm: find the share of each.
5. If $\frac{5}{7}$ of a sum of money be decreased by $\frac{2}{9}$ of the sum, the remainder will be \$62 more than $\frac{20}{41}$ of the remainder: find the sum.
6. A train going 25 miles an hour starts at 1 o'clock P.M. on a trip of 280 miles; another going 37 miles an hour starts from the same place at 12 min. past 4 o'clock P.M.: when and where will the former be overtaken?
7. Divide \$540 between George, Henry, and Fred, giving George 30% more than Fred's share, and Henry \$80 more than the other two together.
8. By selling my cloth at \$1.26 a yard, I gain 11 cents more than I lose by selling it at \$1.05 a yard: what would I gain by selling 800 yards at \$1.40 a yard?

PAPER VI.

Ex. 6.—If 27 men build 32 yards of fencing in 64 hours, how long will it take 36 men to build 108 yards of the same kind of work?

First, looking at the amount of work done, 108 yards will require $\frac{108}{32}$ of the time required for 32 yards.

That is: $\frac{108}{32}$ of 64 hrs.

Again, considering the number of men, 36 men will require $\frac{27}{36}$ of the time required by 27 men.

That is: $\frac{27}{36}$ of $\frac{108}{32}$ of 64 hrs. = 162 hrs.

☞ In all problems such as the preceding, every corresponding pair of terms is formed into a proper or an improper fraction, and the different fractions thus formed are multiplied together.

---

1. How many square yards in an acre? What part of a sq. yd. must be added to $\frac{5}{18}$ sq. ft. + $\frac{7}{18}$ sq. in. to produce 2 sq. ft. 67 sq. in.?
2. Multiply 40 and 15625 millionths by 1632 ten-thousandths.
3. If $\frac{3}{7}$ of the sum of a certain number and its seventh part, be added to the original number, the result will be 292: what is the number?
4. Divide $6000 between C and D, giving D $1800 more than 3 times the amount C receives.
5. A train leaves Toronto for Collingwood, 95 miles, at 13 miles an hour; another leaves Collingwood for Toronto at the same time, but goes 23 miles an hour: when will they be just 5 miles apart?
6. A room is 4 yds. 2 ft. wide, $5\frac{1}{3}$ yds. long, and 8 ft. high: find the cost of papering the walls with paper 24 in. wide at 6c. a yd.
7. A and B receive the same salary; A saves $\frac{2}{7}$ of his, but B spends $36 a month more than A, and is $30 in debt at the end of 5 months: what does each spend a month?
8. If 12 men in 24 days of 9 hrs. each build 192 cu. ft. of wall, how many men will be required to build 320 cu. ft. of wall in 27 days of 10 hrs. each?
9. A grocer drew off ·45 of a cask of syrup, then 20 gallons, and it was found to want ·65 of being full: how many gallons would it contain?

ANSWERS.

EXERCISE 1.—Page 3.

1. 4; 11; 26; 37; 45; 68; 77; 59; 80; 92.
2. 1, 9; 10, 99; 23, 32; 39, 93; 19, 91; 17, 71; 78, 87; 28, 82.
3. 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47.
   63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78;
   17, 16, 15, 14, 13, 12, 11;
   47, 46, 45, 44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33.
4. 15; 64.
5. 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64. 65, 66, 67, 68, 69, 70, 71, 72, 73, 74.
6. 68, 86; 67, 76; 78, 87.

EXERCISE 2.—Page 5.

1. 117; 311; 511; 575; 899; 400; 69; 507; 860; 410; 909; 787.
2. 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119.
3. 790, 791, 792, 793, 794, 795, 796, 797, 798, 799;
   180, 280, 380, 480, 580, 680, 780, 880, 980;
   607, 617, 627, 637, 647, 657, 667, 677, 687, 697;
   700, 710, 720, 730, 740, 750, 760, 770, 780, 790.
4. 999; 100.
5. 689, 698, 869, 896, 968, 986.
6. 39 (which is the same as 039), 93, 309, 390, 903, 930.
7. 98; 11.
8. (1) 7, 70, 700. (2) 68, 86, 608, 680, 806, 860. (3) 899, 989, 998. (4) 777.

EXERCISE 3.—Page 8.

1. 9048; 5007; 43659; 536302; 405313; 5043037; 64792; 356097; 9345027; 80056; 9090900; 83007; 9090; 375867799; 11071; 6008704.

2. 9999, 1000; 99999, 10000; 999999, 100000; 9999999, 1000000.
3. 3^{072}, 3^{172}, 3^{272}, 3^{372}, 3^{472}, 3^{572}, 3^{672}, 3^{772}, 3^{872}, 3^{972}.
4. 7^{530}; 3^{057}.
5. 8700, 7800, 8070, 7080, 7008, 8007; 8700 greatest; 7008 least.

EXERCISE 4.—Page 11.

1. 13, XIII.
2. 17, XVII.
3. 19, XIX.
4. 26, XXVI.
5. 38, XXXVIII.
6. 44, XLIV.
7. 97, XCVII.
8. 150, CL.
9. 280, CCLXXX.
10. 738, DCCXXXVIII.
11. 844, DCCCXLIV.
12. 1200, MCC.
13. 87, LXXXVII.
14. 6000, $\overline{\text{V}}$M.
15. 1500, MD.
16. 11000, $\overline{\text{X}}$M.
17. 888, DCCCLXXXVIII.
18. 7592, $\overline{\text{V}}$MMDXCII.
19. 4711, M$\overline{\text{V}}$DCCXI.
20. 52, LII.
21. 39, XXXIX.
22. 43, XLIII.
23. 67, LXVII.
24. 91, XCI.
25. 1881, MDCCCLXXXI.
26. 27, XXVII.
27. 49, XLIX.
28. 73, LXXIII.
29. 68, LXVIII.
30. 84, LXXXIV.
31. 97, XCVII.
32. 110, CX.
33. 550, DL.
34. 740, DCCXL.
35. 990, CMXC.
36. 1600, MDC.
37. 50005, $\overline{\text{L}}$V.
38. 318, CCCXVIII.
39. 796, DCCXCVI.
40. 1096, MXCVI.
41. 25000, $\overline{\text{XXV}}$.
42. 59300, M$\overline{\text{LX}}$CCC.
43. 87040, $\overline{\text{LXXXV}}$MMXL.

EXERCISE 5.—Page 13.

(*a*) 1. 116,234. One hundred and sixteen thousand, two hundred and thirty-four.
2. 65,231. Sixty-five thousand, two hundred and thirty-one.
3. 20,703. Twenty thousand, seven hundred and three.
4. 71,005. Seventy-one thousand and five.
5. 3,104. Three thousand, one hundred and four.
6. 48,000. Forty-eight thousand.

7. 60,029. Sixty thousand and twenty-nine.
8. 141,120. One hundred and forty-one thousand, one hundred and twenty.
9. 101,207. One hundred and one thousand, two hundred and seven.
10. 68,978. Sixty-eight thousand, nine hundred and seventy-eight.
11. 72,020. Seventy-two thousand and twenty.
12. 80,001. Eighty thousand and one.
13. 857,000. Eight hundred and fifty-seven thousand.
14. 91,029. Ninety-one thousand and twenty-nine.
15. 7,640. Seven thousand, six hundred and forty.
16. 800,900. Eight hundred thousand, nine hundred.
17. 2,568,242. Two million, five hundred and sixty-eight thousand, two hundred and forty-two.
18. 1,008,003. One million, eight thousand and three.
19. 212,375,647. Two hundred and twelve million, three hundred and seventy-five thousand, six hundred and forty-seven.
20. 609,003,588. Six hundred and nine million, three thousand, five hundred and eighty-eight.
21. 897,856,846. Eight hundred and ninety-seven million, eight hundred and fifty-six thousand, eight hundred and forty-six.

(*b*) 1. 9. Nine.
2. 200. Two hundred.
3. 60,002. Sixty thousand and two.
4. 700,000,000. Seven hundred million.
5. 230,000,060. Two hundred and thirty million, and sixty.
6. 81,501,007,012. Eighty-one billion, five hundred and one million, seven thousand and twelve.
7. 30,000,000,000,603. Thirty trillion, six hundred and three.
8. 700,080,000,000,000. Seven hundred trillion and eighty billion.
9. 8,007,014,010. Eight billion, seven million, fourteen thousand and ten.
10. 15,000,018,207,000,081. Fifteen quadrillion, eighteen billion, two hundred and seven million and eighty-one.

(*c*) 1. 60,701,892. Sixty million, seven hundred and one thousand, eight hundred and ninety-two.

2. 50,607,801. Fifty million, six hundred and seven thousand, eight hundred and one.
3. 600,000. Six hundred thousand.
4. 49,000,000. Forty-nine million.
5. 593,006,070,500. Five hundred and ninety-three billion, six million, seventy thousand, five hundred.
6. 190,190,001,900. One hundred and ninety billion, one hundred and ninety million, one thousand, nine hundred.
7. 163,194,568. One hundred and sixty-three million, one hundred and ninety-four thousand, five hundred and sixty-eight.
8. 3,050,050,183. Three billion, fifty million, fifty thousand, one hundred and eighty-three.
9. 5,000,204. Five million, two hundred and four.
10. 594,900. Five hundred and ninety-four thousand, nine hundred.
11. 12,000,012. Twelve million and twelve.
12. 2,007,980,134. Two billion, seven million, nine hundred and eighty thousand, one hundred and thirty-four.

(*d*) 1. 18,000.
2. 2,060,153.
3. 60,060,060,060.
4. 60,200,500.
5. 402,348,213,020.
6. 78,640,000,006,016.
7. 6,542,000,025.
8. 6,542,000,025.
9. 402,348,213,020.
10. 5,008,949.
11. 200,300,800.
12. 29,599,000,601.
13. 4,000,558,240,070.
14. 32,001,343,404.
15. 555,777,669.
16. 806,070,005,206.

(*e*) 1. 19. Nineteen.
2. 21. Twenty-one.
3. 10. Ten.
4. 45. Forty-five.
5. 65. Sixty-five.
6. 64. Sixty-four.
7. 79. Seventy-nine.
8. 85. Eighty-five.
9. 110. One hundred and ten.
10. 119. One hundred and nineteen.
11. 100. One hundred.
12. 114. One hundred and fourteen.
13. 160. One hundred and sixty.

14. 190. One hundred and ninety.
15. 260. Two hundred and sixty.
16. 290. Two hundred and ninety.
17. 629. Six hundred and twenty-nine.
18. 811. Eight hundred and eleven.
19. 950. Nine hundred and fifty.
20. 1,259. One thousand, two hundred and fifty-nine.
21. 76. Seventy-six.
22. 1,869. One thousand, eight hundred and sixty-nine.
23. 5.193. Five thousand, one hundred and ninety-three.
24. 17. Seventeen.
25. 2,222. Two thousand, two hundred and twenty-two.
26. 3,500. Three thousand, five hundred.
27. 450. Four hundred and fifty.
28. 48. Forty-eight.
29. 536. Five hundred and thirty-six.
30. 1,794. One thousand, seven hundred and ninety-four.
31. 10,116. Ten thousand, one hundred and sixteen.
32. 381. Three hundred and eighty-one.
33. 50,999. Fifty thousand, nine hundred and ninety-nine.
34. 1,001,612. One million, one thousand, six hundred and twelve.
35. 5,470. Five thousand, four hundred and seventy.
36. 265. Two hundred and sixty-five.
37. 3,627. Three thousand, six hundred and twenty-seven.
38. 19. Nineteen.
39. 54. Fifty-four.
40. 402. Four hundred and two.
41. 536. Five hundred and thirty-six.
42. 85. Eighty-five.
43. 18. Eighteen.
44. 77. Seventy-seven.
45. 67. Sixty-seven.
46. 164. One hundred and sixty-four.
47. 135. One hundred and thirty-five.
48. 149. One hundred and forty-nine.
49. 1.019. One thousand and nineteen.
50. 653. Six hundred and fifty-three.
51. 100,099. One hundred thousand and ninety-nine.
52. 5,559. Five thousand, five hundred and fifty-nine.
53. 560,000. Five hundred and sixty thousand.
54. 31,500. Thirty-one thousand, five hundred.

55. 59,344. Fifty-nine thousand, three hundred and forty-four.
56. 15,749. Fifteen thousand, seven hundred and forty-nine.
57. 1,003,090. One million, three thousand and ninety.
58. 6,749. Six thousand, seven hundred and forty-nine.
59. 1,525,479. One million, five hundred and twenty-five thousand, four hundred and seventy-nine.
60. 1,882. One thousand, eight hundred and eighty-two.

EXERCISE 7.—Page 21.

(*a*) 1. 11 loads.
2. 12 cents.
3. 12 days.
4. 14 cherries.
5. 16 apples.
6. 14 sheep.
7. 13 dollars.
8. 14 trees.
9. 16 cents.
10. 21 plums.
11. 14 ducks.
12. 19 cents.
13. 13 books.
14. 16 cents.
15. 13 chickens.
16. 9 apples.
17. 17 letters.
18. 12 books.
19. 23 dollars.
20. 16 dollars.
21. 26 cents.
22. 14.
23. 15 cents.
24. 17 birds.
25. 17 cents.
26. 15 panes.
27. 16.
28. 19 pears.
29. 18.
30. 26 cents.
31. 8898 acres.

(*b*) 1. 85.
2. 79.
3. 58.
4. 58.
5. 89.
6. 364.
7. 893.
8. 7795.
9. 8889.
10. 7799.
11. 9979.
12. 6759.
13. 9978.
14. 8999.
15. 6647.
16. 4999.
17. 6999.
18. 77969.
19. 848699.
20. 148798.
21. 8199398.
22. 899969.
23. 9999999.
24. 3747655.

EXERCISE 8.—Page 26.

1.
2. 2435.
3. 799.
4. 678.
5. 568.
6. 9848.
7. 28172.
8. 2598.
9. 2584.
9. 22168.
10. 164918.
11. 3037.
12. 3161.
13. 21286.
14. 23940.
15. 93545.
16. 1245901.
17. 197351.
18. 196395.
19. 154970.
20. 2035342.
2. 1384473.
3. 145406.
4. 1670551.
5. 193372.
6. 4523181.
7. 96379092.

8. 1739183.
9. 134762.
10. 969754.
11. 1825941.
12. 10965339.
13. 1564442.
14. 108594353.
15.
1. 9339190.
2. 11178170.
3. 10306156.
4. 10670291.
16. 4289 trees.
17. $2844.
18. 679 sheep.
19. $9212.
20. 1904.
21. 2153.
22. $7549
23. 3619.
24. 6575.
25 $32331.
26. $11425.
27. 1821.
28. 75922.
29. 339466 men.
30. $13640.
31. $764.
32. $10766.
33. $13025.
34. $2114, $2935, $10897.
35. $141252.
36. 37199.
37. 39351 miles.
38. 154691.
39. 13740178.
40. 1369000920.
41.
1. 16665.
2. 444.
3. 53328.
42. 375540.
43. 60 dahlias.
44. 466 cents.
45. 131 cents.
46. $2538.
47. 96 years.
48. 140 yards.

EXERCISE 10.—Page 37.

1. 142.
2. 1512.
3. 6251.
4. 5021.
5. 1440.
6. 4112.
7. 3201.
8. 5310.
9. 2411.
10. 2412.
11. 1120.
12. 3732.
13. 52221.
14. 852556.
15. 603054.
16. 864201110.
17. 10035174.
18. 5162142.
19. 30640.
20. 57172.
21. 43242.
22. 44211.
23. 32134.
24. 56314.
25. 33662.
26. 24570.
27. 22625.
28. 13432.
29. 43220.
30. 12442.
31. 10853.
32. 25468.
33. 31064.
34. 606453.
35. 64124.
36. 37001.
37. 205443.
38. 601.
39. 35002.
40. 46150.
41. 70046.
42. 10644.
43. 12 marbles.
44. 111 quail.
45. $4410.
46. 213 bushels.
47. $211.
48. 230 sheep.
49. $2560.
50. $1404.
51. $2453.
52. 4003 yards.
53. 4213 dollars.
54 3336 dollars.
55. 24337 dollars.
56. 144422 dollars
57. 843 dollars.
58. 1360 sheep.
59. 513 geese.
60. 202 bushels.
61. 11602 barrels.
62. 14432 dollars.
63. 561092 dollars

EXERCISE 11.—Page 41.

(*a*) 1. 83.
2. 445.
3. 221.
4. 1086.
5. 313.
6. 2384.
7. 3335.
8. 24465.
9. 185173.
10. 1147.
11. 618.
12. 909.
13. 949.
14. 3745.
15. 2757.
16. 41830.
17. 581949.
18. 495829.
19. 383969.
20. 194959.
21. 491978.
22. 261636.
23. 294928.
24. 680929.
25. 281939.
26. 490909.
27. 492929.
28. 391616.
29. 17359.
30. 432099.

(*b*) 1. 276.
2. 67.
3. 4004.
4. 664.
5. 3316.
6. 34943.
7. 5498.
8. 17571.
9. 18654.
10. 23017.
11. 57921.
12. 19238.
13. 37486.
14. 111530.
15. 591203.
16. 666667.

(*c*) 1. 409095.
2. 274850.
3. 707970092.
4. 9610.
5. 896.
6. 51306.
7. 60964492.
8. 5499 13964.
9. 46924.
10. 75757.
11. 56300.
12. 699901.
13. 5200.
14. 9688.
15. 75999.
16. 90014.
17. 145129.
18. 254999.
19. 319527.
20. 663367.
21. 427165.
22. 587979.
23. 758451.
24. 823011.
25. 900829.
26. 6898220.
27. 19542.
28. 459611.
29. 31229.
30. 698.
31. 996.
32. 459812.
33. 44007.
34. 491693.
35. 62796.
36. 4989050.
37. 1700261.
38. 6634585.
39. 69994.
40. 8974088.
41. 287949.
42. 852642.
43. 899999.
44. 90100199.

(*d*) 1. 64349020.
2. 899450.
3. 11600800.
4. 255350011.
5. 1215.
6. 99999500.
7. 478.
8. 459079260.
9. 40020.
10. 99002992.
11. 615.
12. 776546.
13. 34740.
14. 3877.
15. 2092.
16. 401.
17. 4359999.
18. 1105.

EXERCISE 12.—Page 43.

1. 175 sheep.
2. ———.
3. $7604.
4. $5210.
5. $9161.
6. 285866.
7. 3602.
8. 30.
9. 47.
10. 30.
11. $1462.
12. 128.
13. 4794 miles.
14. 5862 pounds.
15. $221708.
16. 21834 acres.
17. 48805 barrels.
18. 19883.
19. $38094.
20. 13733 feet.
21. $8674.
22. 275 acres.
23. 94760000 miles.
24. $6350.
25. $3087.
26. 487628 feet.
27. 1808.
28. $99635.
29. $24354.
30. 28153 votes.
31. 4491 sq. miles.
32. 3571 feet.
33. 1769.
34. 85 years.
35. 664.
36. 110 years.
37. 2984679.
38. 7925 feet.
39. 7812 feet.
40. 8712 feet.
41. 293 feet.
42. $13667.
43. 6326 acres.
44. 28278 barrels.
45. John 32; James 90.

EXERCISE 14.—Page 55.

(*a*) 1. 246.
2. 268.
3. 446.
4. 492.
5. 556.
6. 990.
7. 2624.
8. 4344.
9. 7258.
10. 7570.
11. 8012.
12. 8616.
13. 284068.
14. 3412648.
15. 7229006.
16. 924356.
17. 3609186.
18. 64823505.
19. 280260.
20. 2115258.
21. 4156984.
22. 871624.
23. 2037468.
24. 2836195.
25. 36880812.
26. 23008960.
27. 5772468.
28. 5105520.
29. 30002504.
30. 1659000.
31. 46832.
32. 182452.
33. 103560.
34. 280210.
35. 42528.
36. 360918.
37. 161525.
38. 115604.
39. 400675.
40. 547476.
41. 86499.
42. 140760.
43. 206739.
44. 115604.
45. 314095.
46. 563136.
47. 113900.
48. 197930.
49. 244182.
50. 52092.
51. 206832.
52. 23504.
53. 59822.
54. 4518.
55. 1230125.
56. 4100832.
57. 4536.
58. 9738.
59. 20200.
60. 219138.
61. 21416.
62. 7846312.
63. 26543.
64. 37704.
65. 39006.
66. 64248.

67. 900867.
68. 9909537.
69. 103235616.
70. 8602968.
71. 8419829.
72. 92618119.
73. 99249624.
74. 37873.
75. 16140.
76. 855547.
77. 10666596.
78. 6666660.
79. 119999988.
80. 44444444.
81. 6909084.
82. 5333328.
83. 576702.
84. 1785.
85. 69160.
86. 18711.
87. 35368.
88. 26243.
89. 118314.
90. 8760.
91. 28140.
92. 10978.
93. 97608.
94. 80432.
95. 98196.
96. 226668.
97. 203236.
98. 3065706.
99. 8363091.
100. 1556112.
101. 5122164.
102. 7732530.
103. 3376821.
104. 7873096.
105. 4384552.
106. 10456704.
107. 1841625.
108. 3765706.
109. 10563091.
110. 4748112.
111. 11122164.
112. 6842529.
113. 733236.
114. 3597781.
115. 5202432.
116. 4591644.
117. 5552640.
118. 3066574.
119. 9585466.
120. 9166404.

(*b*) 1. $585.
2. $1104.
3. $17883.
4. $43074.
5. 26400 feet.
6. 8800 yards.
7. 162 days.
8. 894 days.
9. 3680 bushels.
10. $1715.
11. $2024.
12. 1134 cts.
13. $8964.
14. 3555 miles.
15. 420 men.
16. 21120 feet.
17. 48785 yds.
18. 47136 brls.
19. 40859 shingles.
20. $27912.
21. $76113.
22. 79428 pounds,
23. 316800 inches.
24. 149136 miles.
25. $246736.
26. $24332.
27. $10344.
28. 36960 feet.
29. 14080 yds.
30. 980 pounds.
31. $700.
32. 445 cts.
33. $3572.
34. 3483 miles.
35. $7156.
36. $679.
37. 33804 quarts.
38. 1760 sq. rods.
39. 394992 pence
40. 4015 days.

EXERCISE 15.—Page 61.

(*a*) 1. 161840.
2. 149994.
3. 397714.
4. 1153362.
5. 727608.
6. 322992.
7. 334422198.
8. 184752.
9. 343536.
10. 28434195.
11. 16799905881.
12. 584720181340.
13. 694417836.
14. 1521808704.
15. 3529163131975.
16. 163016.
17. 2921005.
18. 879417.
19. 2869605.
20. 7036293.
21. 9130257

22. 11058.
23. 16799905881.
24. 155572.
25. 45758.
26. 4159296.
27. 998001.
28. 707281.
29. 14130081.
30. 7977489.
31. 3769248.
32. 671180.
33. 268056.
34. 2165268.
35. 2879253.
36. 5169248.
37. 7062272.
38. 7520415.
39. 23065974.
40. 34985162.
41. 42397406.
42. 123614208.
43. 19602.
44. 2563912.
45. 131328.
46. 230850.
47. 31812417.
48. 3379446.
49. 2420880.
50. 4040138.
51. 2738352.
52. 384134.
53. 2145594.
54. 14645612.
55. 16003352.
56. 64421850.

(*b*) 1. 740.
2. 8690000.
3. 4698000.
4. 7698400000.
5. 131141615220.
6. 6477150.
7. 278170200.
8. 31915254.
9. 8009628o6.
10. 4793554242.
11. 36613538100.
12. 31617000.
13. 73865000.
14. 130321000.
15. 1231680000.
16. 110040000.
17. 346949400.
18. 16865628oo.
19. 4946796000.
20. 762294.
21. 8697821.
22. 38214.
23. 765870.
24. 600236.
25. 7281711.
26. 38673498.
27. 67312668.
28. 73818055.
29. 241768.
30. 5118862.
31. 17902976.
32. 15403736.
33. 15704325.
34. 2082600.
35. 271541350.
36. 1508741097.
37. 1587862270.
38. 3654860576.
39. 8198473608.
40. 982275037.
41. 3363731415.
42. 559616.
43. 257460.
44. 14988456.
45. 14925792.
46. 11155248.
47. 182151828.
48. 148644288.
49. 1724573025.
50. 8828716566.
51. 601344.
52. 29784450.
53. 7364101944.
54. 104598273[illegible].
55. 118605858c.
56. 15920205.

(*c*) 1. 53207.
2. 182688.
3. 3628800.
4. 202301.
5. 36153036.
6. 163536.
7. 30026997000.
8. 461041.
9. 7546009.
10. 2890000.
11. 12321795560.
12. 628331.
13. 195942000.
14. 104329.
15. 5536076.
16. 328347675.
17. 594992.
18. 18810224.
19. 6332592.
20. 4903524000.
21. 502705700.
22. 1418516800.
23. 415143630.
24. 500; 6[illegible]; 1800; 4700; 2205; 8720[illegible]; 54010; 8080.
25. 725 cents.
26. 90 cents.
27. 975 cents.
28. 60000 cents.
29. 3800 cents.
30. 228240 cents.

(*d*) 1. 4695 bush.
2. 432 panes.
3. 7200 bush.
4. 46797 dollars.
5. 1494 dollars.
6. 38988 dollars.
7. 8125 potatoes.
8. 1547 days.
9. $264958.
10. 404914 lbs.
11. 186516 yards.
12. $7640136.
13. 202640 lbs.
14. $204763.
15. $44975.
16. $121552.
17. 104160 sheets.
18. $299720.
19. 203322 yards.
20. 8480 yards.
21. 68520 feet.
22. 94848000 miles.
23. $71240.
24. $10800.
25. 14688 miles.
26. $1757924.
27. 5280 feet.
28. 79000 pounds.
29. $43250.
30. 2323024 pages.
31. 88486 sheep.
32. 137970 miles.
33. 32166 dollars.
34. 29730 dollars.
35. 541112 dollars.
36. 492729 dollars.
37. 3112135 cents.
38. 128690 dollars.
39. 1730368 dollars.
40. 1074354 days.
41. 5451 dollars.
42. 14689 dollars.
43. 734481 dollars.
44. 228984 men.
45. 273249 yards.
46. 980019 pounds.
47. 372480 plants.
48. 1358112 letters.
49. 349860 pounds.
50. 63360 feet.
51. 40824 plums.
52. 89232 rails.
53. 10248 soldiers.
54. 108 miles.
55. 14712 men.
56. 462160 feet.
57. 595680000 miles
58. 291214 cents.
59. 68400 cents.

EXERCISE 16.—Page 69.

1. 26.
2. 21.
3. 32.
4. 33.
5. 20.
6. 0.
7. 26.
8. 20.
9. 18.
10. 10.
11. 36.
12. 97.
13. 48.
14. 102.
15. 210.
16. 159.
17. 300.
18. 700.
19. 80.
20. 0.
21. 41.
22. 570.
23. 92.
24. 1100.
25. 94.
26. 314.
27. 460.
28. 48.
29. 85.
30. 74.

EXERCISE 19.—Page 82.

1. 312.
2. 431.
3. 342.
4. 132.
5. 231.
6. 212.
7. 121.
8. 221.
9. 185.
10. 4321.
11. 1231.
12. 3102.
13. 373.
14. 184.
15. 153.
16. 252.
17. 232.
18. 142.
19. 131.
20. 121.
21. 4623.

22. 34.
23. 44.
24. 43.
25. 42.
26. 52.
27. 62.
28. 42.
29. 71.
30. 92.
31. 42.
32. 51.
33. 2032.
34. 2042.
35. 1051.
36. 1052.
37. 1054.
38. 252.
39. 489.
40. 218.
41. 324.
42. 224.
43. 135.
44. 155.
45. 105.
49. 2052.
47. 2025.
48. 1507.
49. 4686.
50. 1237.
51. 1213.
52. 1207.
53. 1109.
54. 10856.
55. 3439.
56. 5416.
57. 7432966.
58. 4105.
59. 14469.
60. 5092103.

EXERCISE 20.—Page 83.

1. 913—2.
2. 14516—4.
3. 28823—2.
4. 5786—3.
5. 52954—3.
6. 82304—6.
7. 30411—2.
8. 79965—4.
9. 339758—1.
10. 877022—3.
11. 835581—1.
12. 560054—6.
13. 1046518—2.
14. 485109—3.
15. 41243—5.
16. 124715—3.
17. 814744—6.
18. 772927—4.
19. 411869—2.
20. 561436—6.
21. 680078—1.
22. 731040—4.
23. 558129—5.
24. 844484.
25. 568345—5.
26. 537720—5.
27. 822311—5.
28. 384597—2.
29. 710443.
30. 589684—9.
31. 115993.
32. 944202—10.
33. 3749195—9.
34. 192850—5.
35. 297691—3.
36. 389751—9.
37. 779108—10.
38. 490939—2.
39. 9007 times.
40. 7008 times.
41. 6703.
42. 6324.
43. 8232 times.
44. 5531.
45. 4205 times.
46. 6132 times.

EXERCISE 21.—Page 85.

1. 731—6.
2. 8317—4.
3. 61—92.
4. 73—1.
5. 9—7312.
6. 839—16.
7. 5137—12.
8. 7—12934.
9. 3—92.
10. 37—214.
11. 74—321.
12. 306.
13. 30000.
14. 60—60600.
15. $108.62.
16. $3.12.
17. $4610.
18. 70 cents.
19. 8610 bills.
20. 73107 dollars.

EXERCISE 22.—Page 85.

1. 2448 bricks.
2. 23527 dollars.
3. 52126 dollars.
4. $15967204.
5. 15037 acres.
6. $2126495.
7. 33514 acres.
8. 2889133 sq. yds.
9. 5292 hills.
10. 7968516.
11. 781341 59.
12. 30784 dollars.
13. 16; $24.88.
14. 2172; 1448; 1086.
15. 173; 248; 1588.
16. 128; 191; 1654.
17. 961; 1922; 3844.
18. 33; 99; 198 gals.
19. 410; 816 lbs.
20. 16; 48; 288 tkts.
21. 81; 48; 87 feet.
22. 23; 92; 276 yds.
23. 27; 36 pounds.
24. 11; 33 feet.
25. 85 barrels.
26. 1264 dollars.
27. 1812 dollars.
28. 715 barrels.
29. 199 hats.
30. 159 barrels.
31. 52 weeks.
32. 125 dollars.
33. 212 dollars.
34. 99 miles.
35. 213 acres.
36. 151 dollars.
37. 15 dollars.
38. 25 cents.
39. 118 buttons.
40. 3159 Y's; 1053 Z's.
41. 786 dollars.
42. 147 dollars.
43. 18 dollars.
44. 469 coats.
45. 523 barrels.
46. 447 barrels.
47. 1441 months.
48. 255 baskets.
49. 375 dollars.
50. 294 bushels.
51. 6236 dollars.
52. 362 miles.
53. 7625 dollars.
54. 379 gallons.
55. 11500000 miles.

EXERCISE 23.—Page 91.

1. 21.
2. 23.
3. 24.
4. 21.
5. 214.
6. 234.
7. 235.
8. 324.
9. 243.
10. 342.
11. 227.
12. 218.
13. 229.
14. 257.
15. 228.
16. 207.
17. 1205.
18. 1324.
19. 1523.
20. 1407.
21. 1025.
22. 426.
23. 243.
24. 162.
25. 452.
26. 664.
27. 777.
28. 1802.
29. 1945.
30. 4372.
31. 6203.
32. 9216—543.
33. 5679—350.
34. 12474—401.
35. 21175—499.
36. 81987—39.
37. 771849—173.
38. 474536—523.
39. 252384—2534.
40. 201.
41. 134.
42. 785
43. 54.
44. 617.
45. 1190.
46. 243.
47. 2371.
48. 1655037.
49. 34807.
50. 14076.
51. 1011.
52. 463.
53. 7289.
54. 394.
55. 7071.
56. 28c04.
57. 1172.

58. 3645—2867.
59. 4629.
60. 346.
61. 19214—542.
62. 34045—1098.
63. 1343—1652.
64. 12207—445.
65. 529—2228.
66. 9830291—7000.
67. 5006284.
68. 4000059.
69. 5748362.
70. 2779458—5888.
71. 6438192—73401.
72. 13786926.
73. 537—3.
74. 7973—23.
75. 927—80.
76. 34959—716.
77. 27959—333.
78. 14959—966.
79. 285340—4322.
80. 76360—33498.

EXERCISE 24.—Page 95.

1. 31207—1.
2. 16695—17.
3. 519225—11.
4. 20914—11.
5. 167988—17.
6. 33721—18.
7. 263945—18.
8. 17529—37.
9. 128795—17.
10. 145264—23.
11. 105911—25.
12. 111263—21.
13. 111215—27.
14. 130136—25.
15. $258.
16. 86.
17. 213.
18. $864.
19. 436—1.
20. 83—49.
21. 76—30.
22. 7—491.
23. $8600.
24. 31—2946.
25. 3697—2.
26. 327—311.
27. 386—58.
28. $316.
29. 21—320.
30. 10—11127.
31. 179—452.
32. 243553—7.
33. 15.
34. $115.
35. 13—9112.
36. 11—91800.
37. 3463—42.
38. 5—6512.
39. $46.
40. $10000.
41. $72.
42. $360.
43. $8.46.
44. $7500.62.
45. $812.43.
46. $481.99.

EXERCISE 25.—Page 96.

1. $97.
2. $25.
3. $8.
4. 315 pages.
5. 74 days.
6. $35.
7. $4.
8. $237.
9. 336 hams.
10. 365 questions.
11. 2765 days.
12. 83 years.
13. $237.
14. 3864.
15. 77 battalions.
16. 646.
17. 1872.
18. 894 pounds; 312 pounds.
19. 23 hhds.
20. $13.
21. 24 trips; 71.
22. $197.
23. 425 miles.
24. 131.
25. 47 cartridges.
26. 5280 feet.
27. 24964.
28. 1097 bales.
29. 23 pounds.
30. 256 balls.
31. 3906250.
32. 792.

EXERCISE 26.—Page 99.

1. $10394.
2. $825.
3. Houses; $1675.
4. 1428.
5. 354, 177, 118.
6. 207.
7. 306.
8. 156 times.
9. $2015.
10. 146299—1.
11. $1663.
12. 4320 sheets.
13. $182.
14. $2028.78.
15. 132 men.
16. 75 yards.
17. 897.
18. $15444.
19. 91.
20. 119 weeks.
21. 723 gallons.
22. $5.
23. 66 pounds.
24. 792.
25. 9277.
26. 24.
27. 17 books.
28. 119 miles; 595 miles.
29. $1998.
30. $25175.
31. 781 cases.
32. $9.
33. 1903.
34. $1704.
35. $36.
36. 39.
37. $15.
38. 5133.
39. 36405 bush.
40. 162.
41. $3.60.
42. 33 days.
43. $336.
44. $465.50.
45. 68 bush.
46. 8 oranges.
47. 9 hours.
48. 12 days.
49. 5 weeks.
50. 15 days.
51. 37.
52. 217 hats.
53. A owes B; $212.
54. $11080.
55. $3.
56. $2277.
57. $6750.
58. $8.
59. $3.
60. 15.
61. 20 hours.
62. $18.62.
63. 24 months.
64. 31 calves.
65. $70.
66. 3 tons.
67. 21 yards.

EXERCISE 27.—Page 104.

1. 20.
2. 42.
3. 24.
4. 96.
5. 34.
6. 24.
7. 36.
8. 18.
9. 51.
10. 64.
11. 21.
12. 10.
13. 3.
14. 4.
15. 20.
16. 10.
17. 4.
18. 17.
19. 8.
20. 0.

EXERCISE 29.—Page 110.

1. 23.
2. 4.
3. 61.
4. 192.
5. 1.
6. 37.
7. 2.
8. 21.
9. 32.
10. 17.
11. 126.
12. 14 feet.

EXERCISE 31.—Page 115.

1. 90.
2. 360.
3. 12600.
4. 1134.
5. 504.
6. 360.
7. 20160.
8. 480.
9. 438480.
10. 43680.
11. 1330732.
12. 9744.
13. 155232.
14. 72120.
15. 469170.
16. 4149360.
17. 441000.
18. 68640.
19. 1591744.
20. 4200, 210, 168, 140, 120, 105.
21. 300, 150, 100, 75, 60, 30, 15, 6, 3.

EXERCISE 33.—Page 121.

(*a*) 1. $\frac{6}{8}$; $\frac{9}{12}$; $\frac{15}{20}$; $\frac{30}{40}$; $\frac{24}{32}$; $\frac{45}{60}$.
2. $\frac{10}{12}$; $\frac{13\frac{1}{3}}{16}$; $\frac{25}{30}$.
3. $\frac{10}{20}$; $\frac{5}{20}$; $\frac{16}{20}$; $\frac{14}{20}$.
4. $\frac{25}{35}$; $\frac{25}{40}$; $\frac{30}{36}$; $\frac{36}{45}$.

(*b*) 1. $\frac{3}{5}$, $\frac{2}{5}$, $\frac{3}{4}$.
2. $\frac{7}{8}$, $\frac{5}{6}$, $\frac{1}{2}$.
3. $\frac{5}{7}$, $\frac{9}{11}$, $\frac{5}{7}$.
4. $\frac{7}{8}$, $\frac{3}{10}$, $\frac{12}{14}$.
5. $\frac{18}{16}$, $\frac{1}{6}$, $\frac{1}{100}$.
6. $\frac{13}{19}$, $\frac{9}{100}$, $\frac{3}{23}$.
7. $\frac{1}{3}$, $\frac{7}{61}$, $\frac{3}{4}$.
8. $\frac{2}{3}$, $\frac{12}{29}$, $\frac{17}{24}$.
9. $\frac{7}{15}$, $\frac{1}{3}$, $\frac{13}{17}$.
10. $\frac{13}{33}$, $\frac{2}{3}$, $\frac{39}{44}$.
11. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{3}$.

EXERCISE 34.—Page 123.

(*a*) 1. $4\frac{3}{4}$.
2. 7.
3. 7.
4. $6\frac{1}{6}$.
5. $3\frac{4}{5}$.
6. $5\frac{3}{4}$.
7. $5\frac{1}{6}$.
8. $4\frac{7}{8}$.
9. $6\frac{3}{7}$.
10. $7\frac{1}{8}$.
11. $7\frac{1}{2}$.
12. 13.
13. $13\frac{2}{7}$.
14. 17.
15. 4.
16. $2\frac{1}{3}$.
17. $3\frac{3}{4}$.
18. $7\frac{1}{3}$.
19. $4\frac{1}{4}$.
0. $6\frac{3}{4}$.
1. $4\frac{3}{8}$.
22. $4\frac{1}{10}$.
23. 6.
24. $9\frac{3}{4}$.
25. $9\frac{4}{5}$.
26. $12\frac{1}{6}$.
27. 7.
28. 18.
29. $7\frac{1}{9}$.
30. $7\frac{3}{8}$.
31. $8\frac{1}{3}$.
32. 5.
33. $4\frac{4}{5}$.
34. $6\frac{1}{6}$.
35. $4\frac{1}{7}$.
36. 6.
37. 4.
38. $14\frac{3}{8}$.
39. $14\frac{1}{3}$.
40. $17\frac{1}{2}$.
41. $16\frac{2}{3}$.
42. $28\frac{1}{13}\frac{1}{1}$.
43. $31\frac{1}{2}\frac{1}{1}$.
44. $61\frac{29}{276}$.

(*b*) 1. $\frac{20}{3}$.
2. $\frac{37}{4}$.
3. $\frac{27}{4}$.
4. $\frac{51}{5}$.
5. $\frac{19}{2}$.
6. $\frac{17}{5}$.
7. $\frac{25}{6}$.
8. $\frac{31}{8}$.
9. $\frac{17}{3}$.
10. $\frac{31}{4}$.
11. $\frac{42}{5}$.
12. $\frac{59}{6}$.
13. $\frac{43}{4}$.
14. $\frac{59}{7}$.
15. $\frac{46}{6}$.
16. $\frac{58}{11}$.
17. $\frac{76}{7}$.
18. $\frac{97}{8}$.
19. $\frac{100}{9}$.
20. $\frac{89}{9}$.

21. $\frac{20}{12}$.
22. $\frac{59}{7}$.
23. $\frac{47}{6}$.
24. $\frac{77}{8}$.
25. $\frac{53}{5}$.
26. $\frac{89}{7}$.
27. $\frac{59}{4}$.
28. $\frac{50}{3}$.
29. $\frac{114}{5}$.
30. $\frac{227}{7}$.
31. $\frac{228}{5}$.
32. $\frac{95}{4}$.
33. $\frac{37}{7}$.
34. $\frac{131}{8}$.
35. $\frac{209}{12}$.
36. $\frac{630}{11}$.
37. $\frac{734}{9}$.
38. $\frac{249}{2}$.
39. $\frac{1028}{8}$.
40. $\frac{1805}{9}$.
41. $\frac{8794}{7}$.
42. $\frac{46008}{11}$.

(*c*) 1. 12 days; 5 days.
2. 6 ounces; 11 ounces; 9 ounces.
3. $\$\frac{7}{8}$; $\$\frac{1}{11}$.
4. 40 boys; $\$\frac{5}{12}$.
5. $\frac{21}{4}$.
6. $\frac{75}{5}$.
7. $\frac{79}{6}$.
8. $\frac{91}{11}$.
9. $\frac{22}{7}$; $\frac{44}{14}$.
10. $\frac{46}{9}$; $\frac{92}{18}$.
11. $\frac{77}{12}$; $\frac{154}{24}$.
12. $\frac{115}{14}$; $\frac{345}{42}$.
13. $\frac{185}{20}$; $\frac{555}{60}$.
14. $\frac{203}{8}$; $\frac{244}{8}$; $\frac{802}{8}$.
15. $\frac{214}{4}$.

EXERCISE 35.—Page 126.

(*a*) 1. $\frac{10}{12}$, $\frac{8}{12}$, $\frac{7}{12}$, $\frac{6}{12}$.
2. $\frac{30}{20}$, $\frac{16}{20}$, $\frac{7}{20}$, $\frac{6}{20}$.
3. $\frac{33}{36}$, $\frac{18}{36}$, $\frac{16}{36}$, $\frac{2}{36}$.
4. $\frac{80}{45}$, $\frac{27}{45}$, $\frac{24}{45}$, $\frac{20}{45}$.
5. $\frac{69}{72}$, $\frac{60}{72}$, $\frac{45}{72}$, $\frac{16}{72}$.
6. $\frac{114}{120}$, $\frac{110}{120}$, $\frac{105}{120}$, $\frac{80}{120}$.
7. $\frac{675}{300}$, $\frac{270}{300}$, $\frac{156}{300}$, $\frac{60}{300}$.
8. $\frac{48}{8}$, $\frac{26}{8}$, $\frac{5}{8}$, $\frac{4}{8}$.

(*b*) 1. C has most; A least.
2. C most; D least.
3. Charles; Henry; John; James.
4. Riding—longest; Walking—shortest.
5. $\frac{658}{70}$, $\frac{585}{70}$, $\frac{810}{70}$, $\frac{60}{70}$ feet.
6. $\$\frac{2744}{280}$, $\$\frac{1176}{280}$, $\$\frac{980}{280}$, $\$\frac{120}{280}$.

EXERCISE 37.—Page 130.

1. $7\frac{12}{35}$.
2. $32\frac{11}{30}$.
3. $14\frac{5}{6}$.
4. $40\frac{1}{4}$.
5. $3\frac{7}{8}$.
6. $6\frac{3}{28}$.
7. $40\frac{47}{60}$.
8. $36\frac{9}{14}$.
9. $57\frac{7}{12}$.
10. $22\frac{7}{16}$.
11. $39\frac{4}{35}$.
12. $28\frac{41}{60}$.
13. $31\frac{11}{24}$.
14. $624\frac{7}{8}$.
15. $2\frac{2}{45}$.
16. $\frac{2}{25}$.
17. $8\frac{7}{135}$.
18. $\frac{31}{240}$.
19. $5\frac{23}{24}$ acres.
20. $13\frac{3}{20}$ yards.
21. $\$14\frac{61}{72}$.
22. $5\frac{1}{3}$ pounds.
23. $24\frac{3}{4}$ gallons.
24. $\$208\frac{11}{36}$.
25. $\$1\frac{4}{7}$.
26. $16\frac{19}{24}$ miles.
27. $712\frac{1}{4}$ yards.
28. $42\frac{3}{5}$ feet.
29. $82\frac{1}{2}$ yards.
30. $\$96\frac{1}{15}$.
31. $\$29\frac{5}{8}$.
32. $\$19\frac{2}{5}$.
33. $69\frac{70}{83}$.
34. $182\frac{41}{56}$.
35. $1014\frac{44}{63}$.
36. $\frac{5}{48}$.
37. $\$149\frac{1}{48}$.
38. $\$1\frac{37}{48}$.
39. $13\frac{1}{10}$.
40. $88\frac{1}{5}$.
41. $201\frac{47}{48}$ lbs.
42. $\frac{9}{40}$.

EXERCISE 40.—Page 137.

1. $\frac{7}{10}$.
2. $\frac{27}{121}$.
3. $\frac{5}{16}$.
4. $1\frac{3}{17}$.
5. 28.
6. $3\frac{3}{8}$.
7. $238\frac{1}{3}$.
8. $1\frac{4}{11}$.
9. $16\frac{1}{5}$ cts.
10. \$1.30$\frac{4}{5}$.
11. \$1.36$\frac{1}{2}$.
12. \$35$\frac{1}{4}$.
13. 40 miles.
14. $40\frac{1}{4}$.
15. \$224.
16. \$10$\frac{7}{20}$.
17. $38\frac{1}{2}$ cords.
18. \$31.25.
19. \$15.95.
20. 1953 cords.
21. \$10.60$\frac{37}{48}$.
22. \$3$\frac{15}{16}$.
23. 147.
24. \$23$\frac{1}{8}$.
25. \$5734.
26. 42 knots.

EXERCISE 41.—Page 140.

1. $\frac{3}{5}$.
2. $\frac{3}{7}$.
3. $\frac{4}{7}$.
4. $\frac{2}{5}$.
5. $2\frac{1}{2}$ times.
6. 8 times.
7. 6.
8. $3\frac{1}{2}$.
9. $1\frac{9}{16}$.
10. $\frac{3}{7}$ times.
11. $\frac{11}{14}$.
12. $93\frac{22}{51}$ yards.
13. 46 hours.
14. 368 times.
15. $3\frac{57}{65}$.
16. $30\frac{4}{5}$ pounds.
17. $12\frac{7}{12}$ hours.
18. 50 scarfs.
19. 44 bottles; $\frac{10}{11}$ pint.
20. $\frac{19}{110}$ mile.
21. 55 pieces.
22. $9\frac{9}{11}$; $26\frac{2}{11}$; $69\frac{9}{11}$ pounds.
23. $6\frac{1}{4}$ months.
24. 8 days.
25. 400 weeks.
26. $\frac{6}{25}$ mile.

EXERCISE 42.—Page 142.

1. $\frac{9}{20}$.
2. $\frac{12}{35}$.
3. $\frac{25}{42}$.
4. $\frac{8}{15}$.
5. $\frac{21}{32}$.
6. $\frac{10}{27}$.
7. $\frac{1}{3}$.
8. $1\frac{1}{6}$.
9. $8\frac{68}{415}$.
10. $\frac{7}{66}$.
11. $7\frac{7}{10}$.
12. $13\frac{9}{14}$.
13. $8\frac{24}{49}$.
14. $13\frac{4}{7}$.
15. $10\frac{3}{8}$.
16. $1\frac{79}{320}$.
17. 8.
18. $1\frac{1}{2}$.
19. $1\frac{7}{15}$.
20. $11\frac{143}{245}$.
21. $5\frac{50}{81}$.
22. $\frac{2}{7}$.
23. $\frac{19}{51}$.
24. $1\frac{25}{36}$.

EXERCISE 43.—Page 144.

1. $1\frac{5}{12}$.
2. $\frac{67}{604}$.
3. $\frac{2}{7}$.
4. $\frac{2}{9}$.
5. $\frac{1}{7}$.
6. $\frac{4}{5}$.
7. $\frac{6}{11}$.
8. $\frac{24}{35}$ pk.
9. 80 lbs.
10. $19\frac{1}{5}$.
11. \$140.
12. \$30.
13. 32 ounces.
14. \$5$1\frac{1389}{1605}$.
15. \$308.
16. 3 times.
17. 24.
18. $\frac{6}{7}$.
19. \$15.
20. \$35.
21. 5 times.
22. 12 years.
23. $\frac{2}{21}$.
24. $\frac{1}{10}$.

25. $\frac{1}{2}$, $100.
26. $1000.
27. $\frac{1}{36}$, $\frac{1}{13}$.
28. $\frac{1}{11}$.
29. $\frac{4}{21}$.
30. $\frac{1}{8}$.
31. $1\frac{31}{41}$ days.
32. 5 days.
33. $12\frac{3}{4}$ hours.
34. $13\frac{1}{3}$ days.
35. $\frac{1}{9}$ day.
36. 6 days.
37. $\frac{2}{9}$.
38. $13\frac{1}{3}$ min.
39. $24\frac{4}{9}$ min.
40. $1912.50.
41. 60 feet.
42. 150 feet.

EXERCISE 44.—Page 150.

1. $174.55.
2. $149.35.
3. $315.22.
4. $214.96.
5. $1370.68.
6. $1318.95.
7. $2375.85.
8. $6516.03.
9. $1099.52.
10. $830.10.
11. $435.09.
12. $384.87.
13. $235.93.
14. $32.57.
15. $197.38.
16. $9793.61.
17. $8838.81.
18. $28.47.
19. $135.
20. $197.28.
21. $784.
22. $51975.
23. $1564.50.
24. $1804.
25. $125.12.
26. $25839.
27. 86 times.
28. 864 books.
29. 7630 times.
30. $3.16.
31. 16 cts.
32. 581 months.
33. 15 cts.
34. 105 pounds.
35. $35845.35.
36. $92.85.
37. $129.05.
38. $12.12.
39. $530.60.
40. $48.60; 90c.
41. 197 lots.
42. $60.33.
43. $53.55.
44. 32800 pieces.
45. $7.52.

EXERCISE 45.—Page 155.

1. 58; 1092 far.
2. 5616d.; 13435d.
3. 724695 far.
4. £383; £478.
5. 14814 far.
6. £22 5s. 2d.
7. £3 11s. 2d.
8. 3820 far.
9. 203 far.
10. 20163 far.
11. 1. £22 3s. 1d.
    2. £17 6s. $5\frac{1}{4}$d.
12. 21133 sixpences.
13. £46 12s. 3d.
14. 316 boys;
    £3 0s. 11d.
    a. £39 11s. $8\frac{3}{4}$d.
    4. £3001 10s. 10d.

EXERCISE 46.—Page 157.

1. £27 11s. 6d.
2. £18 17s. $8\frac{1}{4}$d.
3. £63 16s. 2d.
4. £66 2s. $1\frac{1}{4}$d.
5. £928 8s. 9d.
6. £4825 5s. $9\frac{3}{4}$d.
7. £99153 10s. $2\frac{1}{4}$d.
8. £66807 9s. 8d.
9. £55862 0s. 4d.
10. £50650 6s. $5\frac{3}{4}$d.

EXERCISE 47.—Page 158.

1. £8 17s. 5d.
2. £9 8s. 10d.
3. £94 15s. 5¾d.
4. £7148 11s. 8½d.
5. £3 2s. 0½d.
6. £6 9s. 7½d.
7. £3109 0s. 11d.
8. £5960 14s. 3¾d.
9. £45 16s. 3d.

EXERCISE 48.—Page 160.

1. £12 19s. 10½d.
2. £37 13s. 5d
3. £6 1s. 2¼d.
4. £449 13s. 9d.
5. £144 7s. 4d.
6. £159 15s. 2¼d.
7. £132 16s. 0½d.
8. £198 3s. 10¾d.
9. £840 11s. 6d.
10. £1134 13s. 1½d.
11. £498 13s. 6d.
12. £155 18s. 8½d.
13. £4630 19s. 5¾d.
14. £76 4s. 9¾d.
15. £15153 12s. 5½d.
16. £1107 8s. 6d.
17. £22 15s. 10d.
18. £52 1s.

EXERCISE 49.—Page 163.

1. £14 9s. 8¼d.
2. £154 1s. 9d.
3. £149 17s. 11d.
4. £73 17s. 8¾d.
5. £3 19s. 7½d.
6. £83 15s. 0¾d.
7. £3 19s. 1½d.
8. £794 18s. 0¼d.
9. £53 12s. 5½d.; £12 13s. 2¼d.; £69 4s. 3½d.; £73 0s. 0½d.
10. £2 10s. 7½d.
11. £4 10s.
12. 7; 11; 16; 108.
13. 8 men.
14. 12 shawls.
15. $\frac{1}{5}$.
16. 121 times.
17. 1200 purses.
18. 13 times; £6 3s. 3½d

EXERCISE 50.—Page 166.

(*b*) 2. 3 oz. 12 dwt. 18 grs.
3. 28968 grs.
4. 173 lbs. 7 oz. 6 dwt. 16 grs.
5. 58321 grs.
6. 3420 grs.
7. 2 lbs. 4 oz. 6 drs. 1 scr.
8. 18465 powders.
9. 584 packages.
10. 5684 oz.
11. 3 cwt. 60 lbs. 7 oz.
12. 3 T. 13 cwt. 59 lbs.
13. 9 T. 8 cwt. 17 lbs.
14. 45 T. 4 cwt. 57 lbs. 2 oz.
15. 107 lbs. 7 oz. 8 dwt. 11 grs.
16. 8 cwt. 31 lbs. 10 oz.
17. 24 T. 1 cwt. 71 lbs.
18. 17 T. 55 lbs.
19. 403 T. 17 cwt. 55 lbs.
20. 689 lbs. 8 oz. 16 dwt.
21. 6 lbs. 10 oz. 10 dwt.
22. 9 cwt. 89 lbs.
23. 890 parcels.
24. 48 ounces.
25. 27 bags.
26. 65 forks; 1 oz. 5 dwt.
27. 2 cwt. $12\frac{5}{8}$ lbs.
28. 24.
29. 50.
30. 5.
31. 2 lbs. 4 oz. 9 dwt. $1\frac{11}{13}$ grs.

32. 51 lbs. 11 oz. 18 dwt. 18 grs.
33. 3 T. 56 lbs. $15\frac{3}{7}$ oz.
34. 3 lbs. 1 oz. 8 dwt. 14 grs.
35. $884\frac{4}{5}$ times.
36. 21 lbs. $6\frac{1}{2}$ oz.; 6 cwt. $72\frac{2}{9}$ lbs.
37. 1 lb. 3 dwt. 16 grs.
38. $\frac{77}{972}$.
39. 10 oz. 9 dwt. $20\frac{49}{23}$ grs.
40. 1 T. 8 cwt. 10 lb. $14\frac{1}{4}$ oz.
41. 18 cwt. $75\frac{7}{11}$ lbs.; 23 T. 5 cwt. $19\frac{377}{532}$ lbs.
42. $\frac{33}{2007}$.

EXERCISE 51.—Page 173.

13. 5 yds. 2 ft. 9 in.
14. 45 yds. 2 ft. 5 in.
15. 1365 inches.
16. $\frac{4}{11}$ of a mile.
17. $22\frac{1}{2}$ cords.
18. 770 fathoms.
19. $56.
20. 9 sq. rd. 13 sq. yds. 1 sq. ft. 34 sq in.
21. 14 inches.
22. $93\frac{1}{3}$ sq. yds.; $16.80.
23. 39 suits.
24. 15587 mi. 7 fur. 33 rds. 2 ft. 3 in.
25. $34\frac{2}{7}$ times.
26. 42 sq. mi. 142 ac. 20 sq. rds. 20 sq. yds. 6 sq. ft. 143 sq. in.
27. 3 ft. $6\frac{2}{3}$ in.
28. 16 feet.
29. 1 ac. 141 sq. rds. 4 sq. yds. 4 sq. ft. 48 sq. in.
30. 19360 sq. yds.
31. 5 mi. 4 fur. 23 rds.
32. 400 ac. 38 sq. rds.
33. $\frac{1}{2}\frac{13}{11}\frac{}{36}$.
34. 45 yds.; $33\frac{1}{3}$ ft.
35. 44880 ft.
36. $\frac{3}{2}\frac{}{5}\frac{}{6}$.
37. 100 acres.
38. 198 ac. $126\frac{2}{3}\frac{}{3}$ sq rds.

EXERCISE 52.—Page 177.

8. 31536000; 31622400; 2592000; 2678400; 2505600; 604800; 86400; 3600 sec.
9. 525600; 527040; 44640; 43200; 44640; 40320; 131040; 132480 min.
10. 18912 seconds.
11. 10 hrs. 41 min. 37 sec.
12. 201 days.
13. 2 wks. 5 da. 7 hrs. 9 min.
14. $\frac{3}{3}\frac{1}{3}\frac{3}{6}$.
15. 5 da. 54 min.
16. 354 days.
17. 10 minutes.
18. $\frac{29}{84}$.
19. 1 wk. 2 da. 4 hrs. 45 min. 59 sec.
20. $\frac{5}{64}$.
21. 5 da. 17 hrs. 22 min.
22. 18 hrs. 1 min. 58 sec.
23. 66 da. 17 hrs. 10 min.
24. 4382 da. 21 hrs. 45 min. $56\frac{2}{5}$ sec.
25. $\frac{480}{13}$; $\frac{1}{39}$; $\frac{1}{2773}$.
26. 6 hrs.; 3 da. 12 hrs.
27. 2 min. $15\frac{5}{7}$ sec.
28. 11 hrs. 54 min. 40 sec.
29. 5 hrs. $42\frac{6}{7}$ min.; $17\frac{1}{7}$ min. to 4 P.M.
30. $4\frac{8}{15}$ min. to 2 P.M.
31. 43 hrs. 47 min. $0\frac{10}{21}$ sec.
32. Saturday, at 10.20 A.M.

EXERCISE 53.—Page 180.

9. 480 dozen.
10. $1091\frac{7}{13}$.
11. 55 pints.
12. 43 bush. 1 pk. 3 qts.
13. 3 hhds. 8 gal. 2 qts. 1 pt.
14. $90.56.
15. 364 gals.
16. $511.87\frac{1}{2}$.
17. 4 loads.
18. $\frac{21}{32}$.
19. 41 gals. 3 qts. 1 pt.
20. $3.80.
21. $1.05.
22. $\frac{1}{3}$.
23. 30 gals.
24. $14\frac{2}{5}$ qts.
25. 36 barrels; 432 dozen.
26. $169\frac{3}{5}$ bush.
27. $922\frac{6}{17}$ bush.
28. 171 bush. 3 qts.
29. $\frac{8}{15}$.
30. $204\frac{4}{46}$ barrels.

EXERCISE 54.—Page 182.

7. 2450 score.
8. 62500 boxes.
9. $720.
10. 80 brls.; $470.40.
11. 1152 books.
12. 300 stones.
13. 115 brls.; $1897.50
14. $\frac{1}{2}$.
15. $416\frac{2}{3}$ doz.
16. $\frac{3}{2}$; $\frac{9}{10}$; $\frac{4}{5}$.
17. $13.20.
18. $83\frac{1}{3}$ reams.
19. $12.
20. $213.16\frac{19}{20}$.
21. 2970 volumes.
22. 185 doz.; $726; 2 ac.

EXERCISE 55.—Page 189.

(*a*) 1. $\frac{99}{200}$; $\frac{3}{400}$; $12\frac{4}{5}$; $68\frac{3}{16}$. 2. $\frac{3}{8}$; $\frac{9}{40}$; $\frac{17}{2500}$; $\frac{5}{16}$. 3. $\frac{19}{20}$; $\frac{7}{8}$; $\frac{7}{16}$; $8\frac{11}{40}$.

(*b*) 1. 65·046 ch.
2. 549·375 ac.
3. ·0003764.
4. 5·6857 ac.
5. 5·937 lbs.
6. 99·96154 miles.
7. ·850955.
8. $108·675.
9. 391·05 ft.
10. ·08.
11. ·15; 3650; 10020.
12. 4·9834; 24·8266; ·6344.
13. ·04026; ·01274; ·00094.

EXERCISE 56.—Page 193.

(*a*) 1. ·625.
2. ·0625.
3. ·85.
4. ·52.
5. ·35.
6. 2·625.
7. ·024.
8. ·0390625.
9. 24·6.
10. ·484375.
11. ·11584.
12. 1·5008.
13. $\dot{0}2\dot{7}$.
14. $\dot{0}8\dot{3}$.
15. $\dot{1}2\dot{3}$.
16. $\dot{2}38095\dot{}$.
17. $\dot{6}2\dot{4}$.
18. $\dot{3}1\dot{9}$.

(*b*) 1. $\frac{6}{9}\frac{5}{0}\frac{5}{9}$.
2. $\frac{8}{11}$.
3. $\frac{5}{37}$.
4. $\frac{7}{75}$.
5. $\frac{324}{1375}$.
6. $\frac{14}{825}$.
7. $\frac{9}{44}$.
8. $\frac{23}{220}$.
9. $\frac{1}{41}$.
10. $\frac{1}{16500}$.

(*c*) 1. $18\cdot\dot{5}\dot{6}$.
2. $\cdot\dot{6}$.
3. $\cdot\dot{1}2\dot{3}$.
4. 8.
5. $7\cdot7\dot{2}$.
6. $7.\dot{3}$

EXERCISE 57.—Page 195.

13. $239.25; $336.
14. $10\frac{1}{2}$d.; $\frac{9}{19}$ gal.; $\frac{3}{4}$ yd.; $2\frac{1}{3}$.
15. $33\frac{1}{3}\%$; 100%; 50%.
16. £50 2s. 1d.; £200 8s. 4d.
17. $103.10; $824.80.
18. 50%.
19. 28%.
20. $26\frac{2}{3}\%$.
21. $110.
22. 50%.
23. 25%.
24. $30000.
25. 19845.
26. 60 cts.
27. $20.
28. $42.87\frac{1}{2}$.
29. 600 lbs. 1st year; 720 lbs. 2nd year.
30. Wife and son, $480 each; daughter, $360.

EXERCISE 58.—Page 200.

1. 9 ft.
2. $15\frac{1}{3}$ ft.
3. 1 ft.
4. 60 sq. yds.; 10 ft.
5. 24 ft. 3 in.
6. $166.88.
7. $31\frac{1}{2}$ in.
8. 12 ft.
9. $9.

EXERCISE 59.—Page 202.

1. $2750.53.
2. $843.
3. £189.
4. $650.10.
5. $13.84.
6. $832.70.
7. $7981.20.

EXAMINATION PAPERS.—Page 203.

PAPER I.

2. $\frac{2}{7}\frac{5}{5}$.
3. $224.
4. $10\frac{9}{10}$ cts.
5. 17.
6. $1200.
7. $\frac{2}{3}\frac{8}{1}\frac{3}{7}$.
8. 288 ac.
9. $3561; $4022.
10. $89.11.

PAPER II.

1. 1854.
2. 14839.
3. CXIV;
   $\bar{X}$MCMLXXXIII.
4. 2880.
5. $2000.
6. 60;
   $2.50.
7. 7 gals.
8. A $360; B $480.
9. 3 times.
10. $18333\frac{1}{3}$.
11. 282 ac.
12. $550.

PAPER III.

1. 197·325.
2. 800;
   200.
3. 70.
4. $5\frac{1}{3}$ ft.
5. $3\frac{1}{8}$ hrs.
6. 21 brls.
7. $7\frac{4}{1}\frac{3}{4}\frac{3}{0}$.
8. $16.

PAPER IV.

1. ·0122.
2. $32\frac{2}{3}$ hrs.
3. $90.70.
4. $68.60.
5. 4 hrs.
6. ·432.
7. $\frac{2}{7}$.
8. A 315 ac.
   B $127\frac{1}{2}$ ac.
   C $127\frac{1}{2}$ ac.

PAPER V.

1. 606100 gals.
2. 1 lb. 3 oz. 8 dwt.
   21 grs.
3. $\frac{7}{11}$.
4. 3410 ac; 3940 ac.
5. $246.
6. 8 min. to 11 P.M.;
   $246\frac{2}{3}$ miles.
7. Henry $31[illegible];
   George $130;
   Fred $100.
8. $240.

PAPER VI.

1. 4840 sq. yds.;
   $\frac{3}{1}\frac{3}{3}$.
2. 6·53055.
3. 196.
4. C $1050;
   D $4950.
5. $2\frac{1}{2}$ hrs.
6. $4.80.
7. A $75;
   B $111.
8. 16 men.
9. 100 gals.